Puff Field Theory

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UC Berkeley and LBNL

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The Sowers Theoretical Physics Workshop 2007

What is String Theory?

Virginia Tech

Based on

- Aki Hashimoto, Sharon Jue, Bom Soo Kim, Anthony Ndirango and OG, "Aspects of Puff Field Theory," [arXiv:hep-th/0702030]
- OG, "A New Lorentz Violating Nonlocal Field Theory From String-Theory," [arXiv:hep-th/0609107]

Introduction

- FRW cosmology: $SO(3,1) \to SO(3)$

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  $\to$ Nonlocality.
- String theory construction?

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  $\rightarrow$ Nonlocality.
- String theory construction?
- $c > 1$?

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[Amelino-Camelia, Ellis, Mavromatos, Nanopoulos and Sarkar, 1998]

$$\frac{\Delta c}{c} < \left| \frac{E}{10^{17}\mathrm{GeV}} \right|, \qquad 1\mathrm{MeV} < E < 17\mathrm{MeV}.$$

[Boggs, Wunderer, Hurley and Coburn, 2003]

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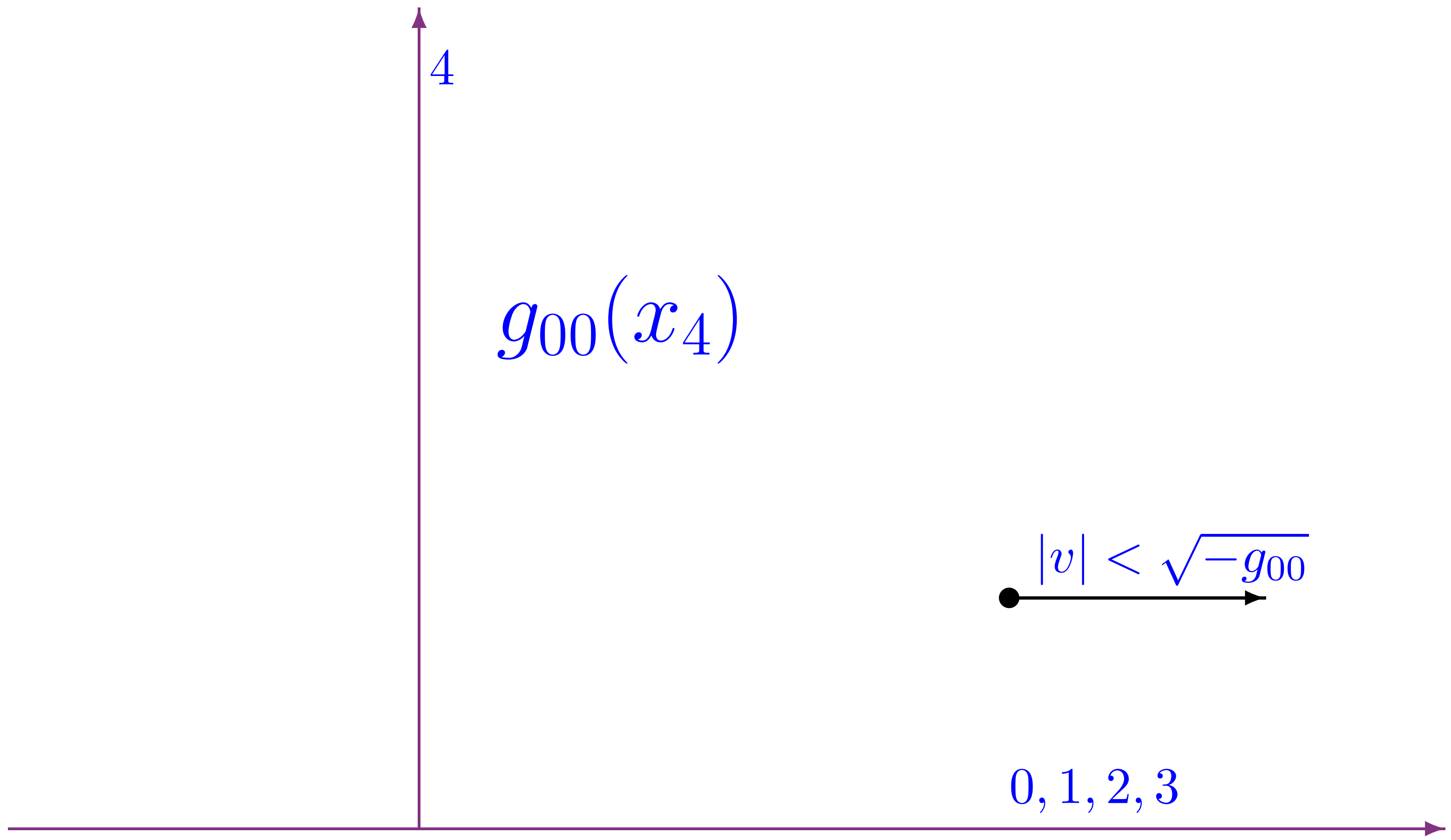
Preserve Lorentz invariant photon dispersion relation.

Lorentz Violation & Extra Dimensions



[Csaki, Erlich and Grojean, 2000]

Nonlocality

- Yang-Mills theory on a Noncommutative $\mathbb{R}^4$ (NCYM)
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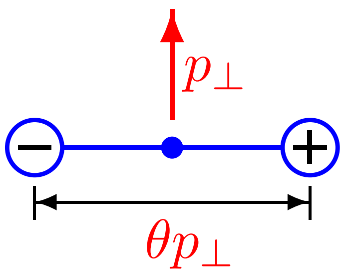

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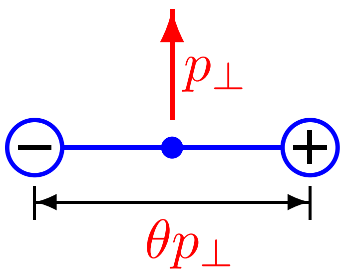

$$\Phi_1 \star \Phi_2 = \left(\exp \frac{i}{2}\theta^{ij}\partial_i^{(1)}\partial_j^{(2)}\right)[\Phi_1\Phi_2]$$

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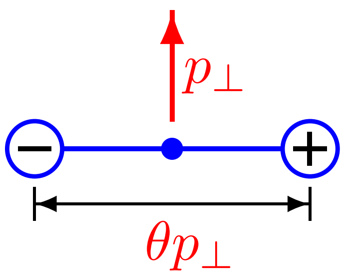

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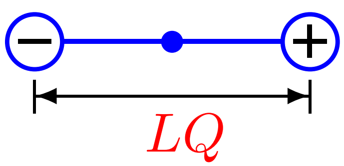
- Dipole theories.

$Q =$ R-charge



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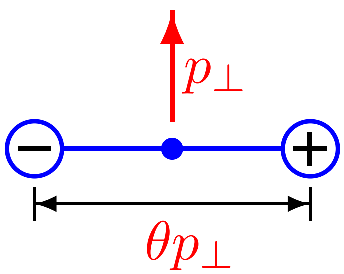

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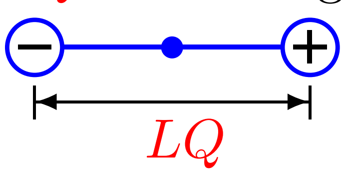

$$D_\mu \Phi(x) = \partial_\mu \Phi(x) - iA_\mu(x - \tfrac{1}{2}LQ)\Phi(x) + i\Phi(x)A_\mu(x + \tfrac{1}{2}LQ)$$

Nonlocality - Volume

- In this talk:

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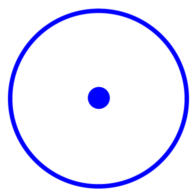

Volume $\propto Q$

Deformations of $N = 4$ SYM

$$L = L_{N=4} + \zeta \mathcal{O}^{(\Delta+4)} + \cdots$$

$$[\zeta] = \text{Length}^{\Delta}$$

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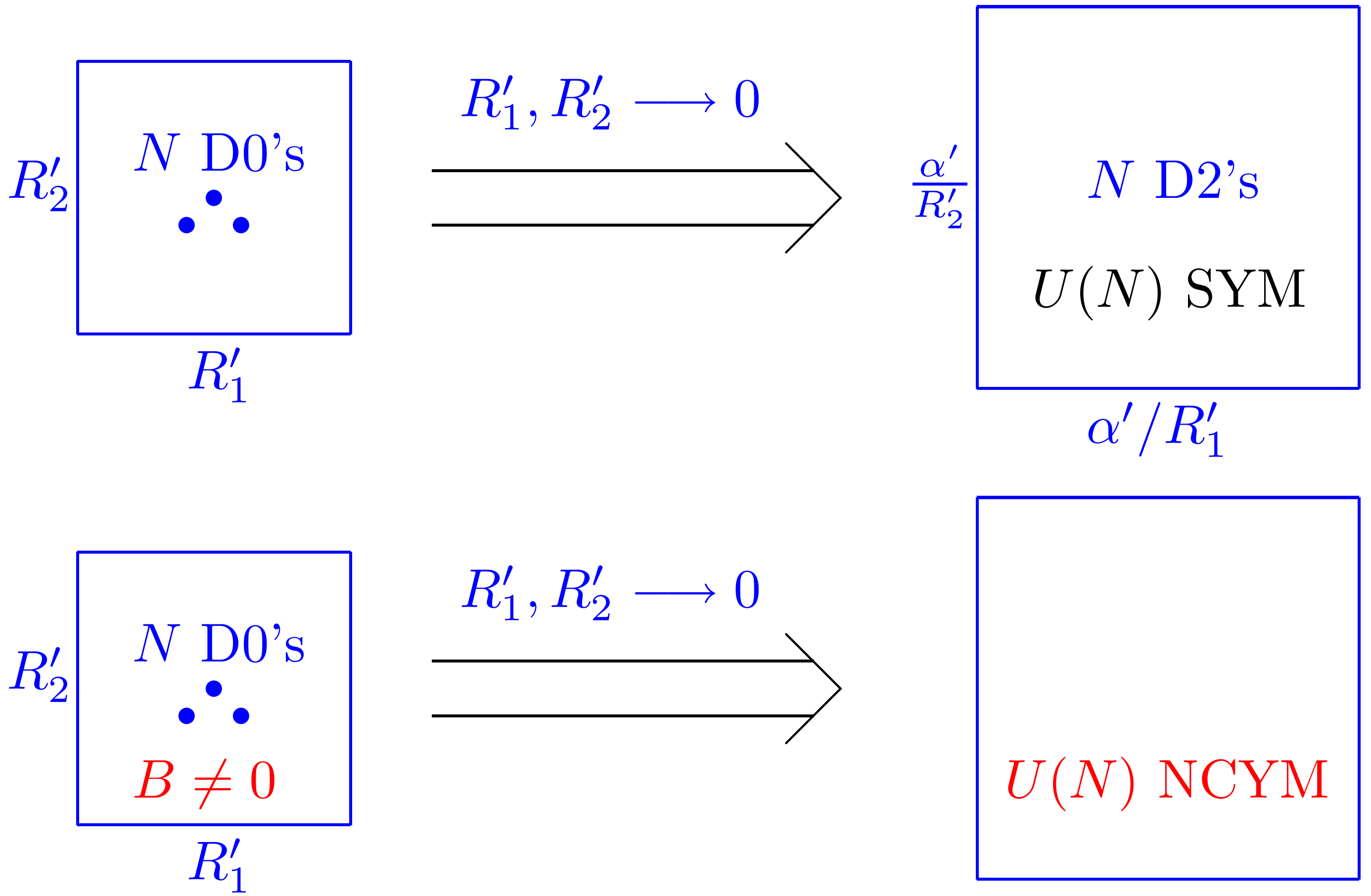
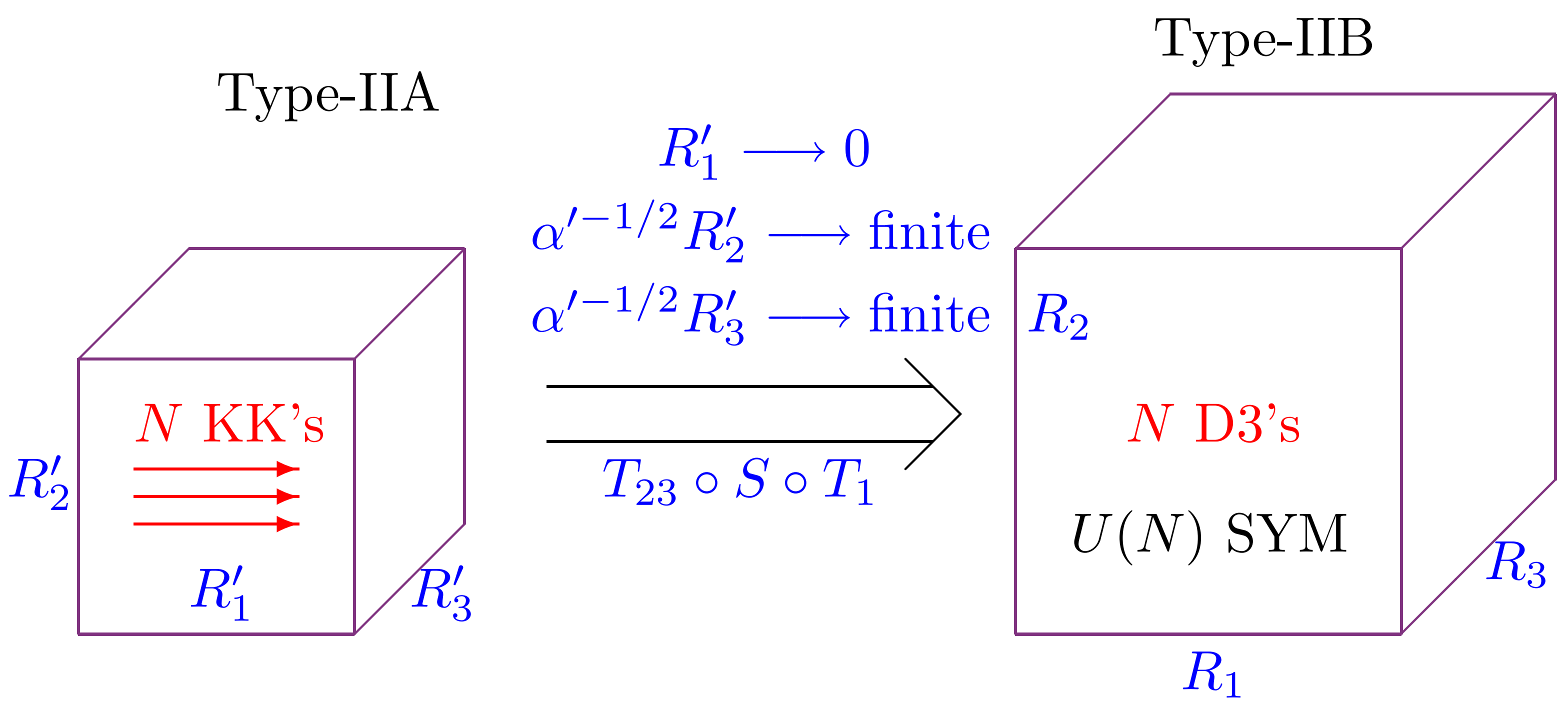
Theory	Deformation $\zeta\mathcal{O}$	Δ	
NCYM	$(1/2{g_{YM}}^2)\theta^{\mu\nu}\,\mathrm{Tr}\,\{F_{\mu\sigma}F_{\nu\tau}F^{\sigma\tau} + F_{\sigma\tau}F^{\mu\nu}F_{\mu\nu} + \cdots\}$	2	$\zeta \to \theta$
Dipole	$(1/2{g_{YM}}^2)L^{\mu}_{IJ}\,\mathrm{Tr}\,\{F_{\mu\nu}(\Phi^I D^\nu \Phi^J + \cdots) + \cdots\}$	1	$\zeta \to L$
This talk	$\cdots\cdots\cdots$	3	

Douglas-Hull construction of NCYM



Our Construction (1)



$$g_s = \alpha' {R'_3}^{-1} {R'_2}^{-1} \qquad R_2 = \alpha'^{\frac{5}{4}} {g_s}'^{\frac{1}{2}} {R'_1}^{-\frac{1}{2}} {R'_2}^{-1}$$

$$R_1 = \alpha'^{\frac{3}{4}} {g_s}'^{-\frac{1}{2}} {R'_1}^{-\frac{1}{2}} \qquad R_3 = \alpha'^{\frac{5}{4}} {g_s}'^{\frac{1}{2}} {R'_1}^{-\frac{1}{2}} {R'_3}^{-1}$$

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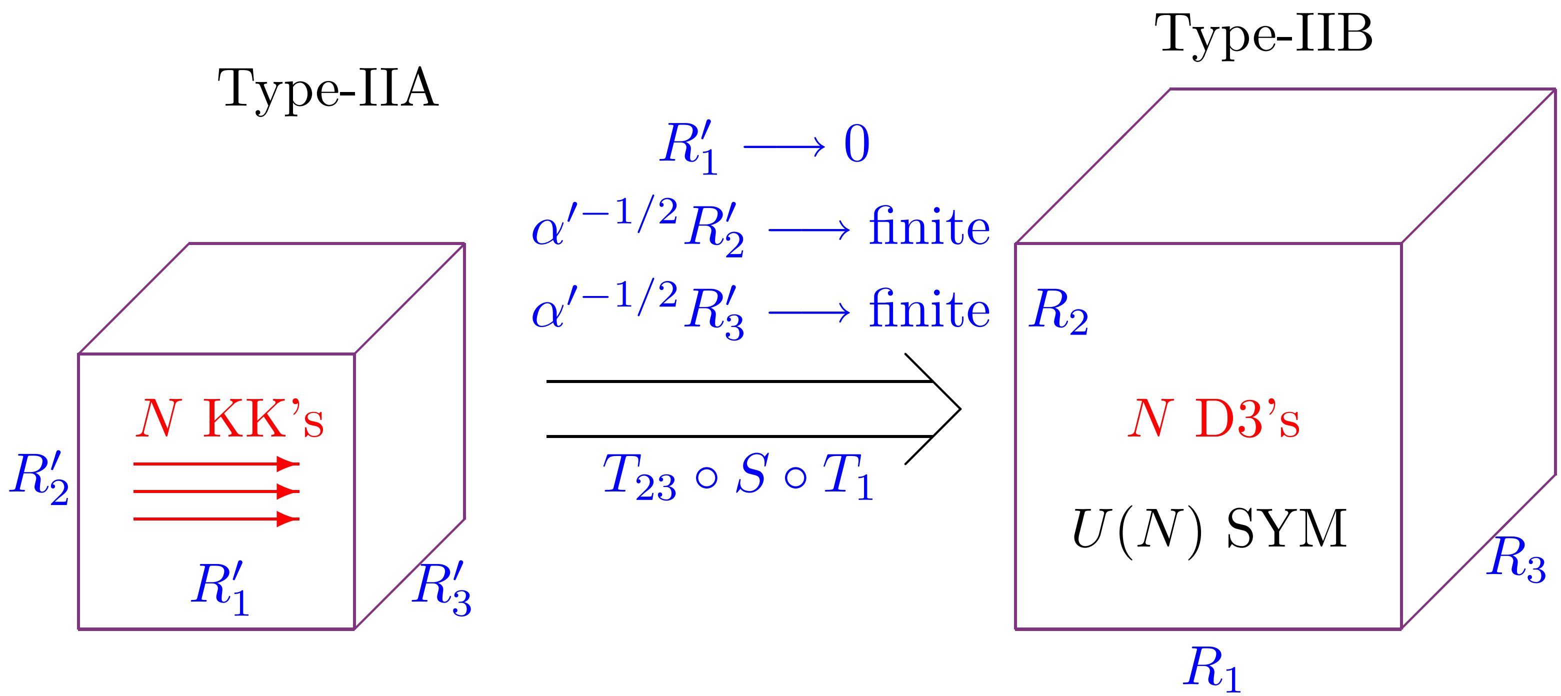
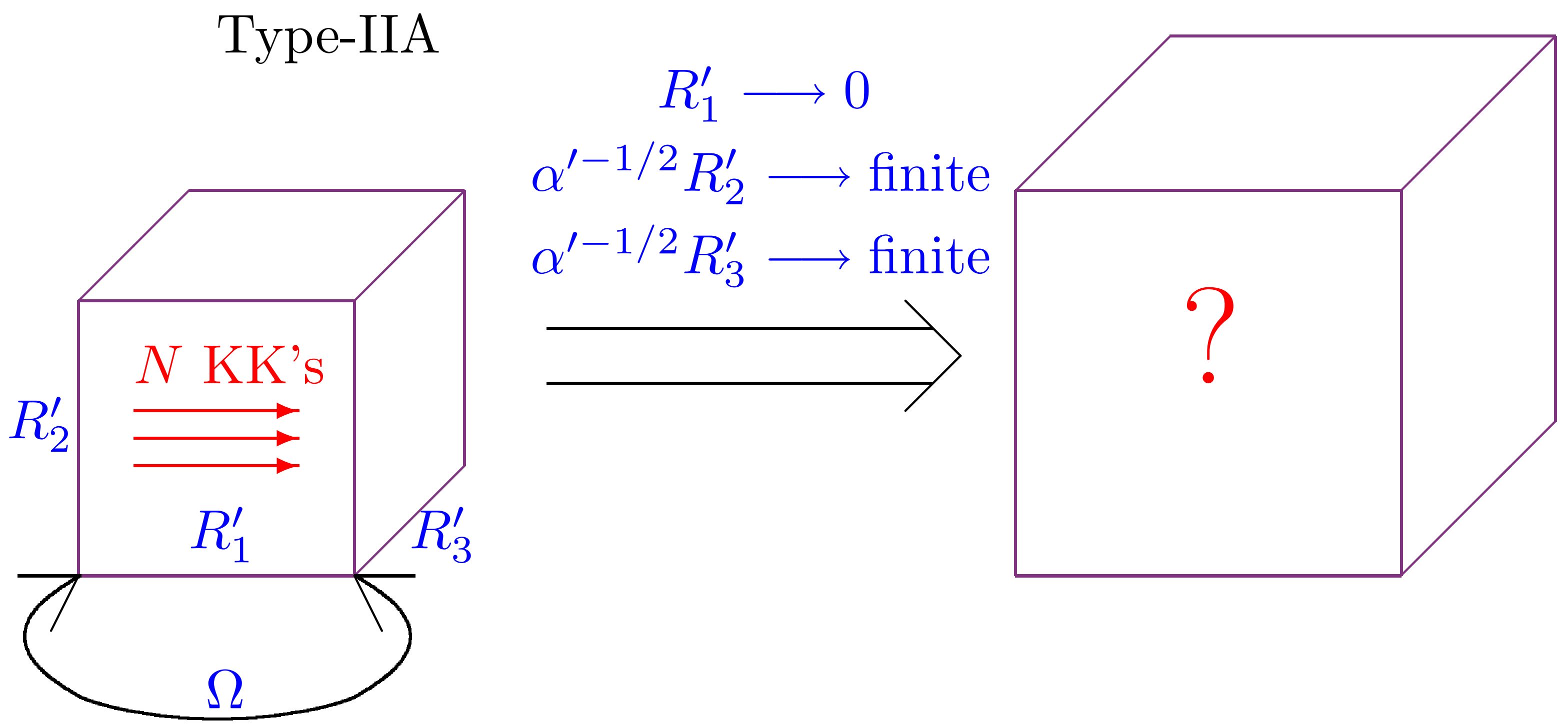


$$g_s \longrightarrow \text{finite} \qquad \alpha'^{-1/2} R_k \longrightarrow \infty, \quad (k = 1, 2, 3)$$

$$R_2/R_1, R_3/R_1 \longrightarrow \text{finite}$$

Our Construction (2)



$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R_1, x_2, x_3, \Omega\vec{y})$$

$$\Omega \in \mathrm{Spin}(6) \qquad \vec{y} \in \mathbb{R}^6$$

Scaling Ω

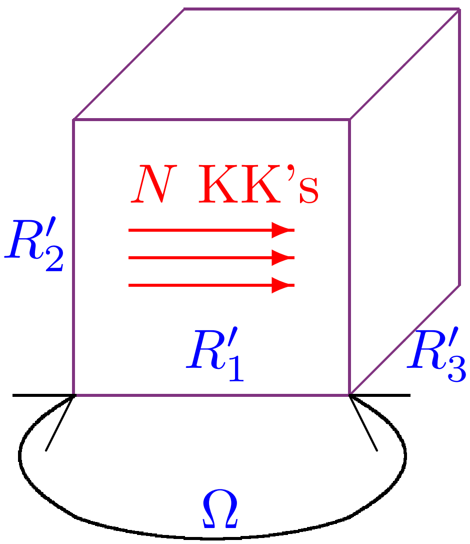
$$\Omega = \exp\left\{\frac{2\pi}{R_1 R_2 R_3}\zeta\right\} \longrightarrow I, \qquad \zeta \to \text{finite}$$

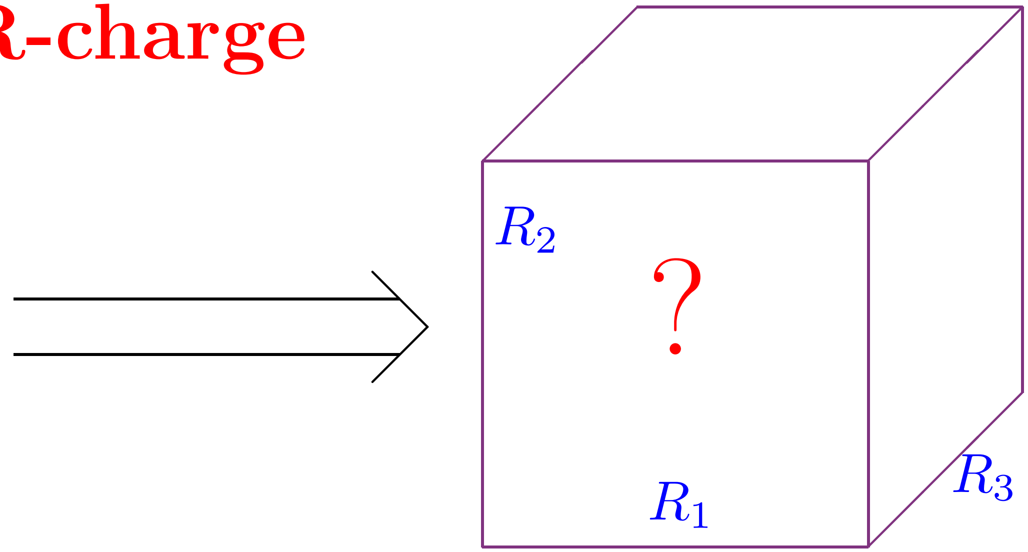
$$\zeta \equiv \begin{pmatrix} & \beta_1 & & & & \\ -\beta_1 & & & & & \\ & & & \beta_2 & & \\ & & -\beta_2 & & & \\ & & & & & \beta_3 \\ & & & & -\beta_3 & \end{pmatrix} \in so(6)$$

SUSY:

$$\begin{array}{ccccccc} \{0\} & \subset & su(2) & \subset & su(3) & \subset & so(6) \\ \mathcal{N}=4 & & \mathcal{N}=2 & & \mathcal{N}=1 & & \mathcal{N}=0 \end{array}$$

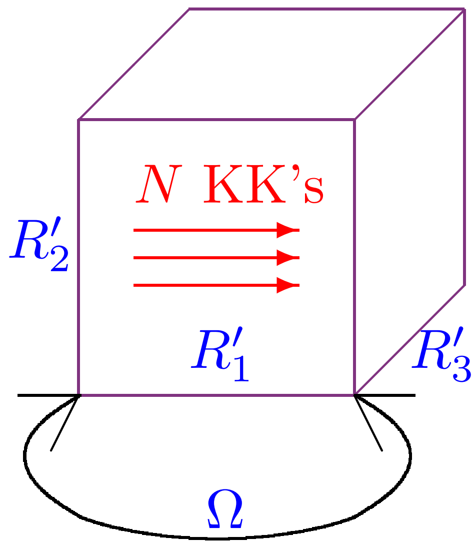
Volume and R-charge

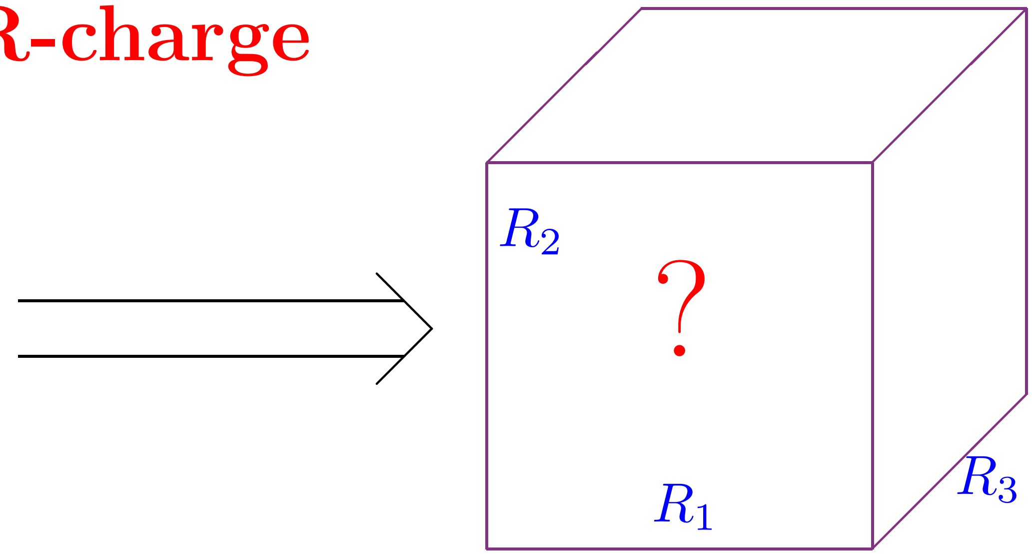




$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R_1', x_2, x_3, \Omega \vec{y})$$

Volume and R-charge

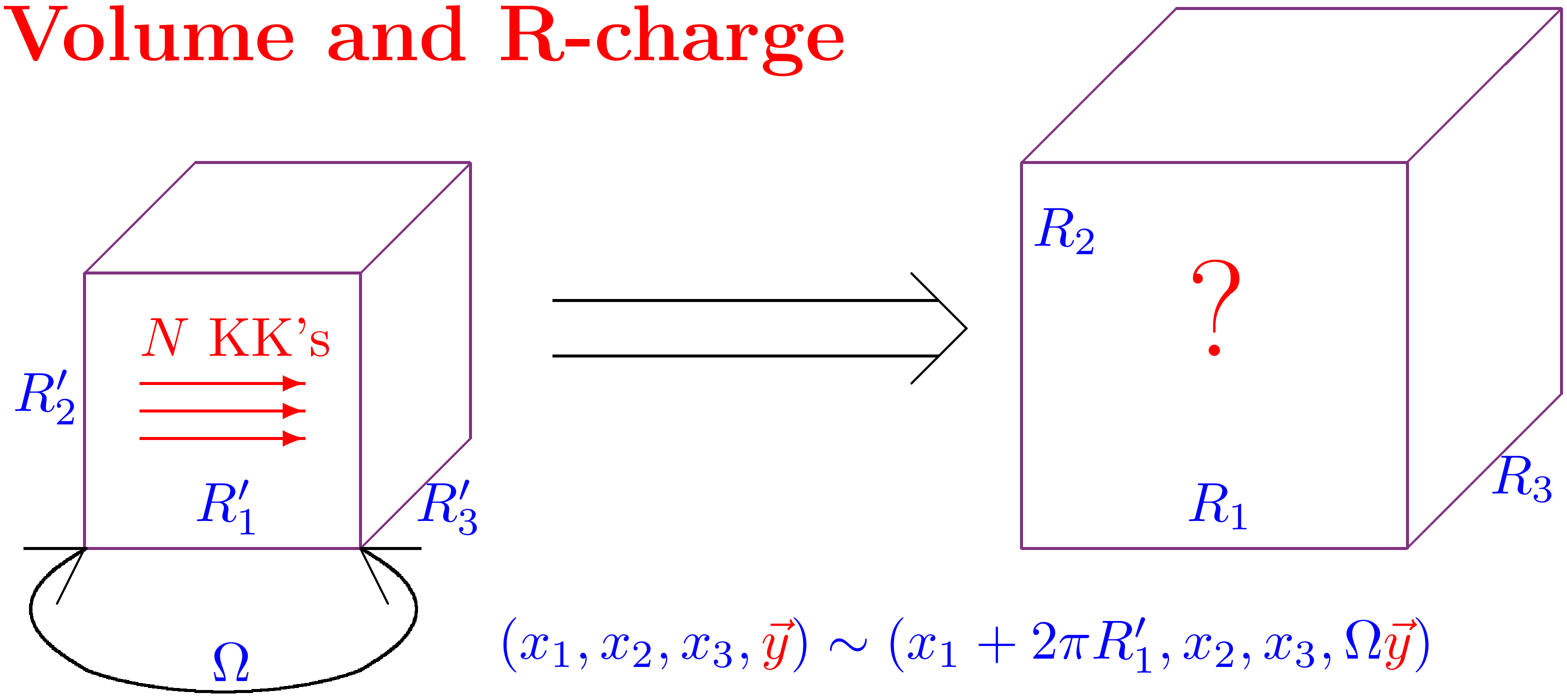




$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R_1', x_2, x_3, \Omega \vec{y})$$

$$z \equiv y_1 + iy_2, \qquad \Omega = \exp \frac{4\pi i \beta}{R_1 R_2 R_3}, \qquad f(x_1 + 2\pi R_1', \Omega z) = f(x_1, z)$$

Volume and R-charge



$$(x_1, x_2, x_3, \vec{y}) \sim (x_1 + 2\pi R'_1, x_2, x_3, \Omega \vec{y})$$

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$$f(x_1, z) = \sum_{p,j} C_{n,j}(|z|) z^{-j} e^{i p x_1 / R'_1} \longrightarrow p \in \mathbb{Z} + \frac{2\beta j}{R_1 R_2 R_3}$$

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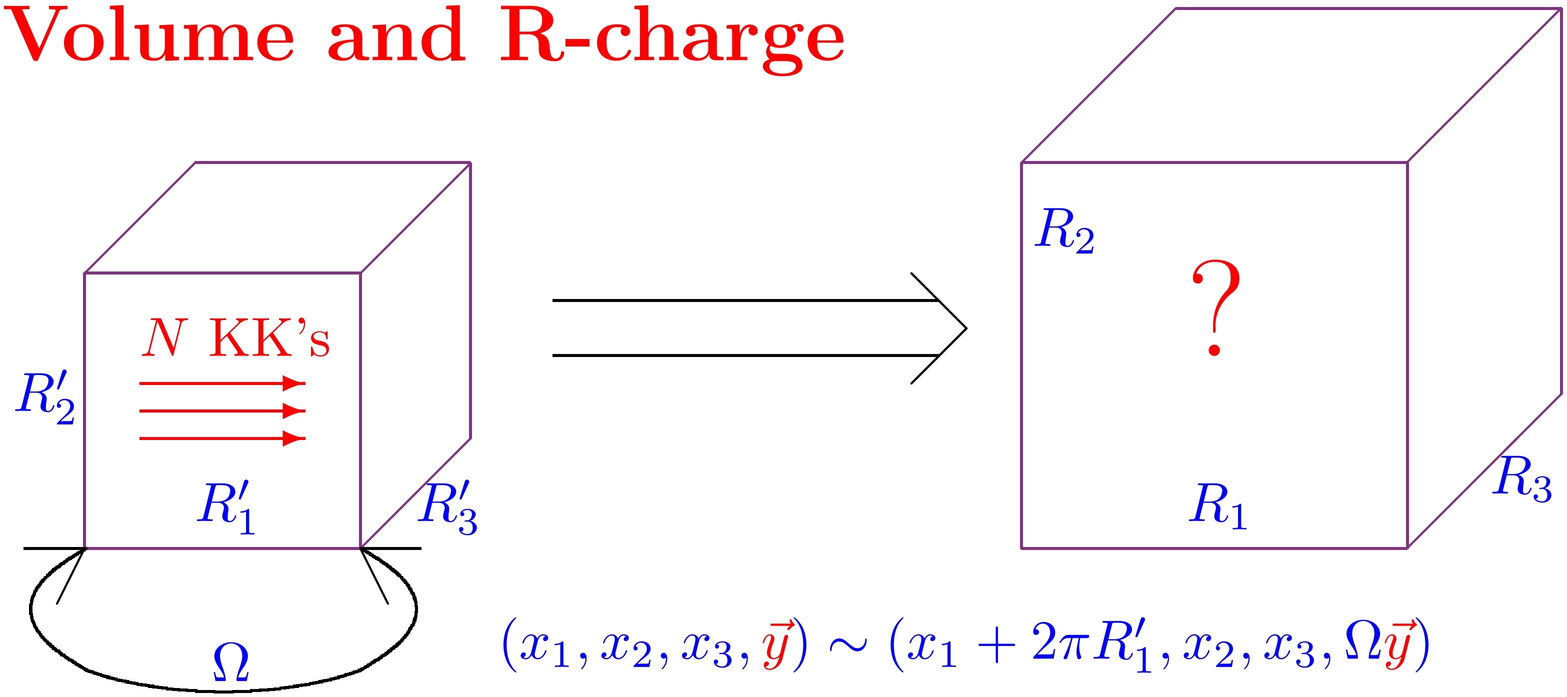

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$$\Rightarrow \boxed{\text{“Vol(D3)”} = (2\pi)^3 (R_1 R_2 R_3 N + 2\beta j)}$$

Electric and Magnetic fluxes

$$V \equiv R_1 R_2 R_3,$$

$$\mathbf{P} \equiv \sum_{i=1}^{3} \frac{k_i}{R_i}\hat{\mathbf{n}}_i, \qquad \mathbf{E} \equiv \sum_{i=1}^{3} \frac{e_i R_i}{2\pi V}\hat{\mathbf{n}}_i, \quad \mathbf{B} \equiv \sum_{i=1}^{3} \frac{m_i R_i}{2\pi V}\hat{\mathbf{n}}_i.$$

$$\begin{aligned} E &= 2j\beta\frac{{M_s}^4}{g_s} + \frac{2\pi^2 V^2}{|NV+2j\beta|}\left(\frac{{g_{YM}}^2}{2\pi}\mathbf{E}^2 + \frac{2\pi}{{g_{YM}}^2}\mathbf{B}^2\right) \\ &\quad +\left|\mathbf{P} - \frac{4\pi^2 V^2}{|NV+2j\beta|}\mathbf{E}\times\mathbf{B}\right|. \end{aligned}$$

$$\mathcal{N} = 2 \Longrightarrow \beta \equiv \beta_1 = \beta_2, \qquad \beta_3 = 0.$$

cf. NCSYM [Ho, Morariu & Zumino, Hofman & Verlinde, Konechny & Schwarz, Pioline & Schwarz,...1998-9].

Supergravity Dual

$$
\begin{aligned}
ds^2 &= \frac{R^2}{r^2} K^{-\frac{1}{2}} \left[dx^2 + dy^2 + dz^2 - \left(dt - \frac{4\pi N}{r^2} \vec{n}^T \zeta d\vec{n} \right)^2 \right] \\
&\quad + \frac{R^2}{r^2} K^{\frac{1}{2}} dr^2 + R^2 K^{\frac{1}{2}} d\Omega_5^2 \\
C_4' &= \frac{\pi N}{r^4} K^{-1} dt \wedge dx \wedge dy \wedge dz \\
&\quad - \frac{\pi N}{g_s {\alpha'}^2 r^6} K^{-1} \vec{n}^T \zeta d\vec{n} \wedge dx \wedge dy \wedge dz,
\end{aligned}
$$

$$
K \equiv 1 + \frac{16\pi^2 N^2}{r^6} \vec{n}^T \zeta^T \zeta \vec{n}
$$

$$
\vec{n} \in S^5, \qquad d\Omega_5^2 = \sum_{I=1}^{6} dn_I^2, \qquad R^4 \equiv 4\pi g_s N {\alpha'}^2
$$

IR limit

$$
\begin{aligned}
ds^2 &= \frac{R^2}{r^2}[dr^2 + dx^2 + dy^2 + dz^2 - dt^2] + R^2 d\Omega_5^2 \\
&\quad + \frac{8\pi N R^2}{r^4}\vec{n}^T \zeta d\vec{n}\, dt + \cdots \\
C_4' &= \frac{\pi N}{r^4} dt \wedge dx \wedge dy \wedge dz \\
&\quad - \frac{\pi N}{g_s \alpha'^2 r^6}\vec{n}^T \zeta d\vec{n} \wedge dx \wedge dy \wedge dz + \cdots
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\end{aligned}
$$

$$
L_{\mathrm{N=4}} \longrightarrow L_{\mathrm{N=4}} + \zeta \mathcal{O} + \cdots
$$

Short multiplets

SYM Operator	desc	SUGRA	dim	spin	$SU(4)_R$
$\mathrm{Tr}\, X^k$	-	$h^\alpha_\alpha \quad a_{\alpha\beta\gamma\delta}$	k	$(0,0)$	$(0,k,0)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\mathrm{Tr}\, F_+ \overline{\lambda}\lambda X^k$	$Q^3\overline{Q}$	$A_{\mu\alpha}$	$k+5$	$(\frac{1}{2},\frac{1}{2})$	$(1,k,1)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\mathrm{Tr}\, F_+ F_-^2 X^k$	$Q^4\overline{Q}^2$	$A_{\mu\nu}$	$k+6$	$(1,0)$	$(0,k,0)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\mathrm{Tr}\, F_+ F_- \overline{\lambda}\lambda X^k$	$Q^3\overline{Q}^3$	$h_{\mu\alpha} \quad a_{\mu\alpha\beta\gamma}$	$k+7$	$(\frac{1}{2},\frac{1}{2})$	$(1,k,1)$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\mathrm{Tr}\, F_+^2 F_-^2 X^k$	$Q^4\overline{Q}^4$	$h^\alpha_\alpha \quad a_{\alpha\beta\gamma\delta}$	$k+8$	$(0,0)$	$(0,k,0)$

taken from [D'Hoker & Freedman, 2003]

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$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathrm{Tr}\, F_+ F_- \bar{\lambda} \lambda$	$Q^3 \overline{Q}^3$	$h_{\mu\alpha} \quad a_{\mu\alpha\beta\gamma}$	7	$(\frac{1}{2}, \frac{1}{2})$	$(1,0,1)$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
$\mathrm{Tr}\, F_+^2 F_-^2 X^k$	$Q^4 \overline{Q}^4$	$h_\alpha^\alpha \quad a_{\alpha\beta\gamma\delta}$	$k+8$	$(0,0)$	$(0,k,0)$

taken from [D'Hoker & Freedman, 2003]

IR-limit

$$L = L_{N=4} + \zeta \mathcal{O}^{(\Delta+4)} + \cdots$$

$$\zeta \mathcal{O} \to \zeta^{\mu}_{AB} T_{\mu\nu} J^{\nu AB} + \zeta^{\mu}_{AB} \epsilon^{ABCDEF} \epsilon_{\mu\nu\sigma\tau} X^C \partial_\nu X^D \partial_\sigma X^E \partial_\tau X^F + \cdots$$

$$J^{\nu AB} \equiv \text{R-current}$$

$$T^{\mu\nu} \equiv \text{Energy-momentum tensor}$$

$$A, B, C, \cdots = 1, \ldots, 6.$$

UV limit

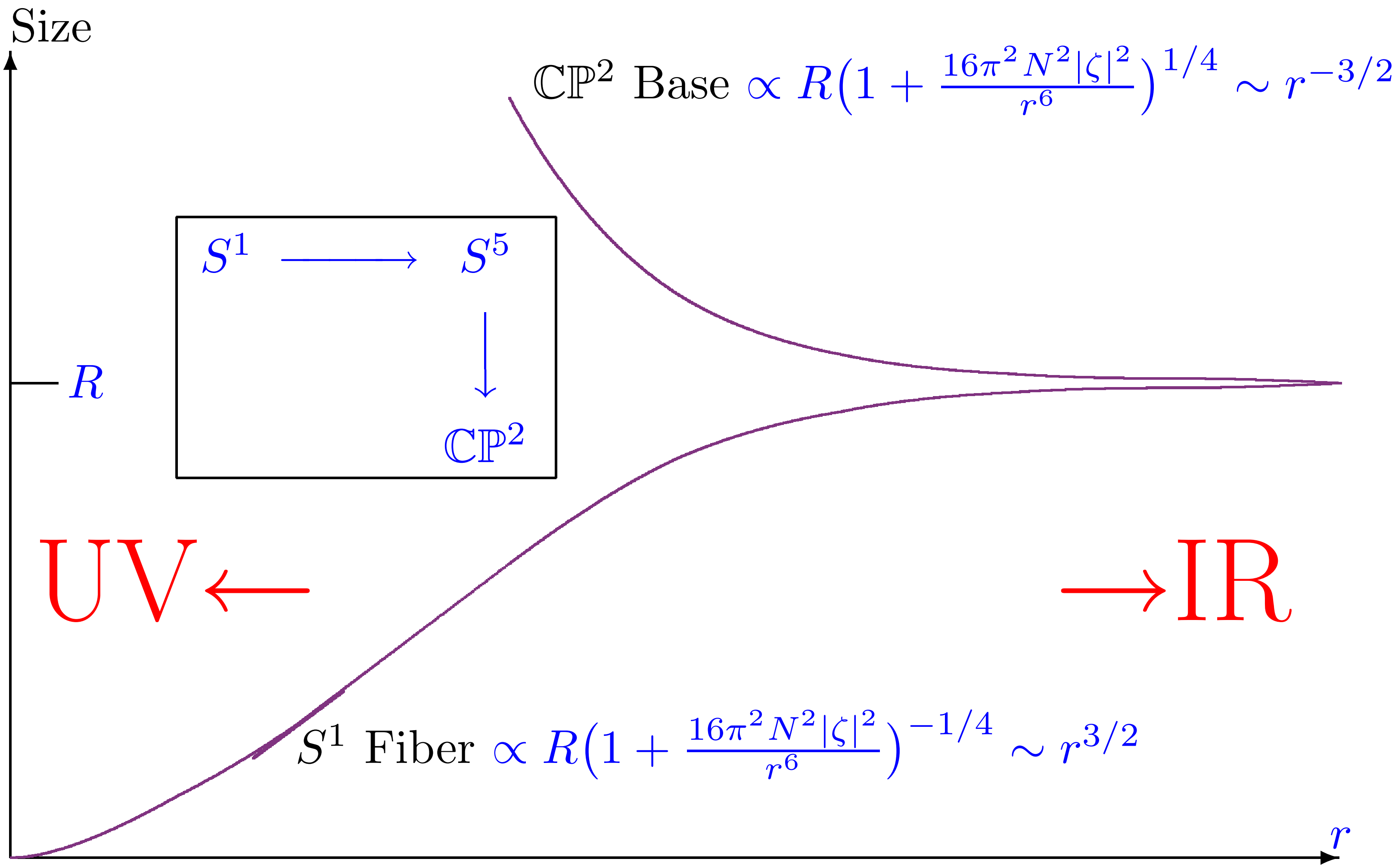
$$\zeta = \begin{pmatrix} & \beta & & & & \\ -\beta & & & & & \\ & & & \beta & & \\ & & -\beta & & & \\ & & & & & \beta \\ & & & & -\beta & \end{pmatrix} \in so(6)$$

$SU(4) \longrightarrow U(3)$, No SUSY.

UV limit (1)

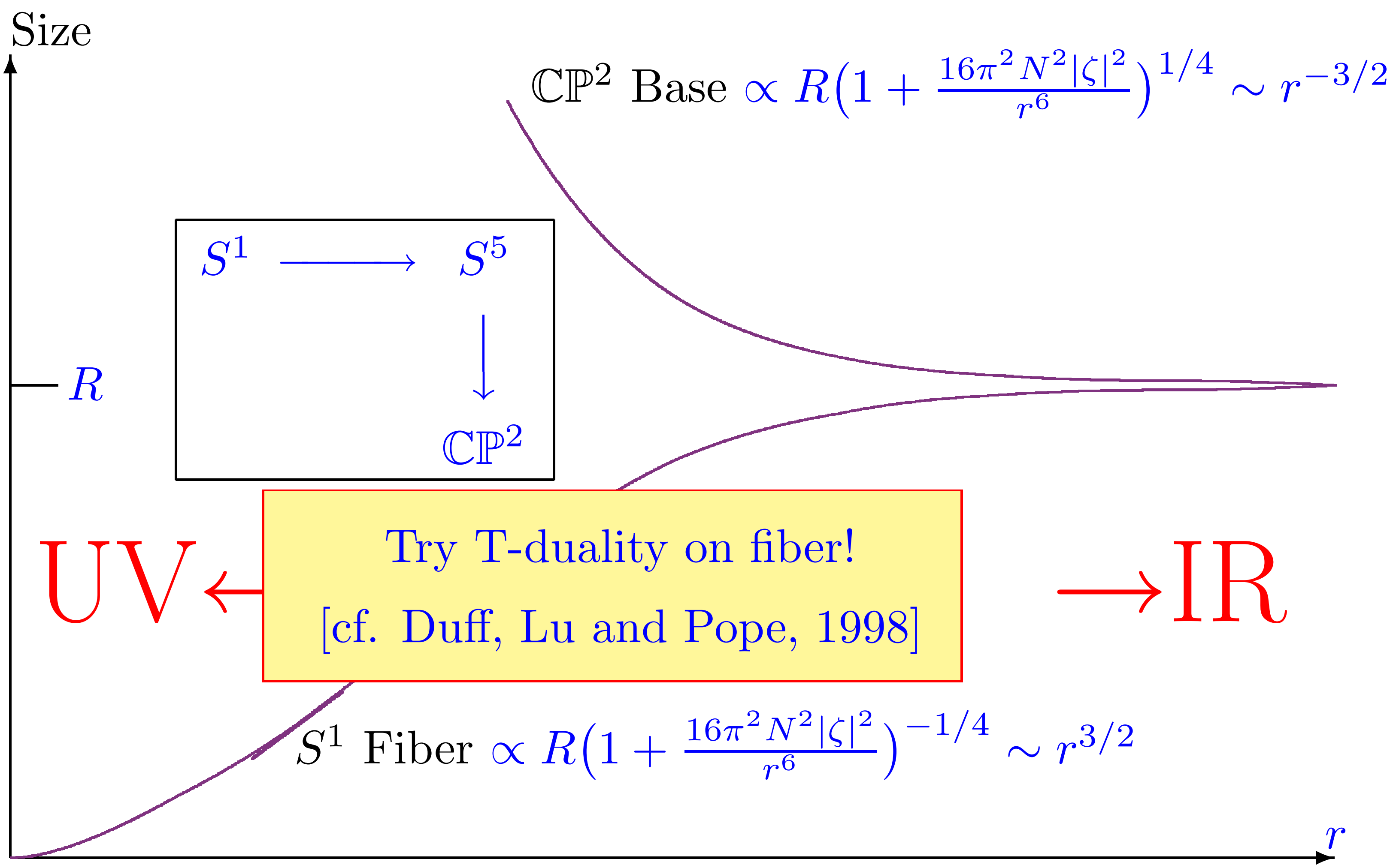
$$\begin{tikzcd} S^1 \arrow[r] & S^5 \arrow[d] \\ & \mathbb{CP}^2 \end{tikzcd}$$

UV limit (1)



Size
R
$\mathbb{CP}^2$ Base $\propto R\left(1+\frac{16\pi^2N^2|\zeta|^2}{r^6}\right)^{1/4} \sim r^{-3/2}$
$S^1 \longrightarrow S^5$
$\downarrow$
$\mathbb{CP}^2$
UV$\leftarrow$
$\rightarrow$IR
S^1 Fiber $\propto R\left(1+\frac{16\pi^2N^2|\zeta|^2}{r^6}\right)^{-1/4} \sim r^{3/2}$
r

UV limit (1)



Try T-duality on fiber!

[cf. Duff, Lu and Pope, 1998]

UV Limit (2)

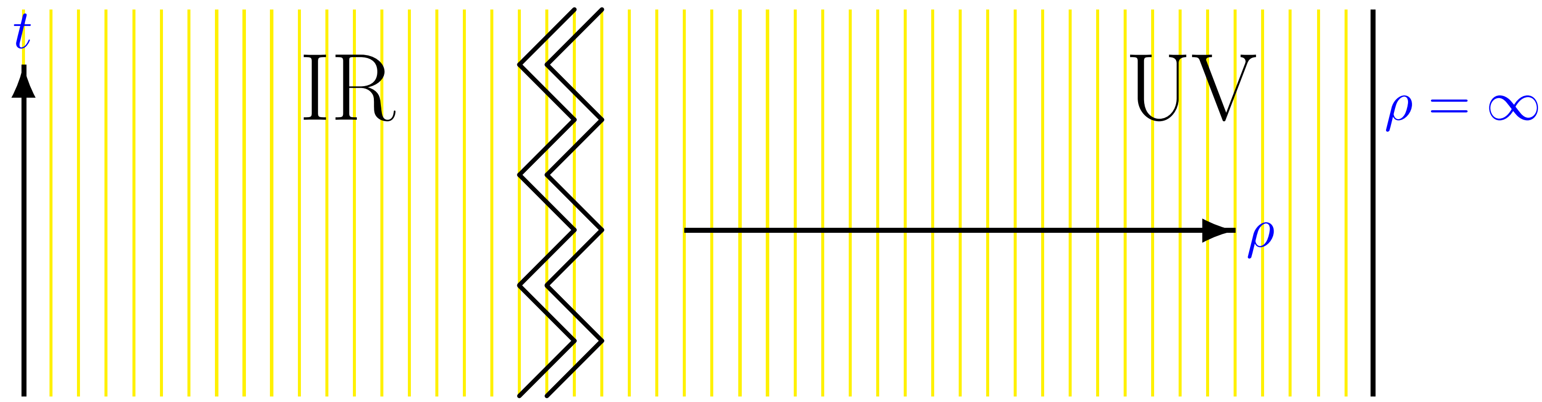
T-duality on fiber: Type-IIB $\Rightarrow$ M.

$$\begin{aligned}
{\ell_P}^{-2} ds_M^2 &= (4\pi N)^{-1/3} {g_s}^{-1} \rho^2 \big[\Delta^{-2/3}(dx^2 + dy^2 + dz^2) - \Delta^{1/3} dt^2\big] \\
&\quad + (4\pi N)^{2/3} \Delta^{1/3} \rho^{-2} d\rho^2 + (4\pi N)^{2/3} \Delta^{1/3} ds_B^2 \\
&\quad + (4\pi N)^{-1/3} \Delta^{1/3} ({g_s}^{-1} d\xi^2 + g_s d\eta^2),
\end{aligned}$$

$$\begin{aligned}
{\ell_P}^{-3} G_4 &= 2\pi N \omega \wedge \omega + 2\omega \wedge d\eta \wedge d\xi \\
&\quad + \frac{2}{3} (4\pi N)^{-2} {g_s}^{-3} \beta d\big(\frac{\rho^6}{\Delta}\big) \wedge dx \wedge dy \wedge dz,
\end{aligned}$$

$$\Delta \equiv 1 + (4\pi N)^{-1} {g_s}^{-3} \beta^2 \rho^6, \qquad (\rho \propto 1/r)$$

Extreme UV Limit



$$\ell_P{}^{-2} ds_M^2 \approx -(4\pi N)^{-2/3} g_s{}^{-2} \beta^{2/3} \rho^4 dt^2$$
$$+(4\pi N)^{1/3} g_s \beta^{-4/3} \rho^{-2} (dx^2 + dy^2 + dz^2)$$
$$+(4\pi N)^{1/3} g_s{}^{-1} \beta^{2/3} (d\rho^2 + \rho^2 ds_B^2) + (4\pi N)^{-2/3} \beta^{2/3} \rho^2 (g_s{}^{-2} d\xi^2 + d\eta^2),$$

$$\ell_P{}^{-3} G_4 \approx 2\pi N \omega \wedge \omega + 2\omega \wedge d\eta \wedge d\xi - 4 g_s{}^3 \beta^{-3} \rho^{-7} d\rho \wedge dx \wedge dy \wedge dz.$$

Extreme UV Limit (2)

$$\boxed{\rho \longrightarrow \infty}$$

- Curvature $\rightarrow$ 0.
- Geodesically complete.
- Redshift in frequency ($g_{00} \longrightarrow \infty$)
- Blueshift in wavelength ($g_{xx}, g_{yy}, g_{zz} \longrightarrow 0$)

Degrees of Freedom

UV regime of PFT – holographically dual to weakly-coupled background (?)

$\boxed{\text{Is the spectrum discrete?}}$

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UV regime of PFT – holographically dual to weakly-coupled background (?)

Is the spectrum discrete?

For particle trajectories with fixed $(E, \vec{p})$:

$$\rho \leq \rho_{\max} \equiv (4\pi N)^{1/6} {g_s}^{1/2} \beta^{-1/3} \left(\frac{E}{|\vec{p}|} \right)^{1/3}$$

Suggests a discrete spectrum!

cf. Little-String-Theory. [Aharony & Berkooz & Kutasov & Seiberg, 1998]

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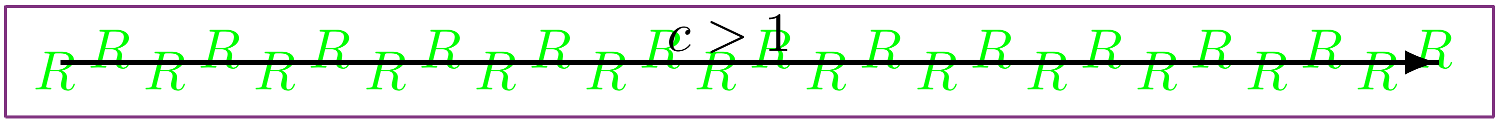

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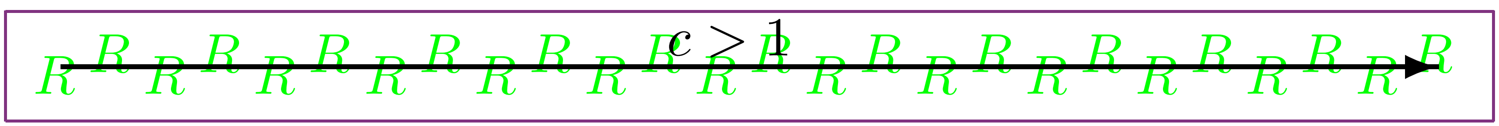

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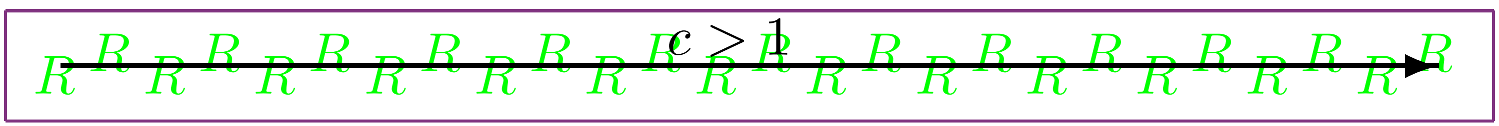

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Trap R-charged particles?

More on $c > 1$

Extreme UV:

$$\begin{aligned}
{\ell_P}^{-2} ds_M^2 \quad &\approx \quad -(4\pi N)^{-2/3} {g_s}^{-2} \beta^{2/3} \rho^4 dt^2 \\
&\quad +(4\pi N)^{1/3} g_s \beta^{-4/3} \rho^{-2} (dx^2 + dy^2 + dz^2) \\
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More on $c > 1$

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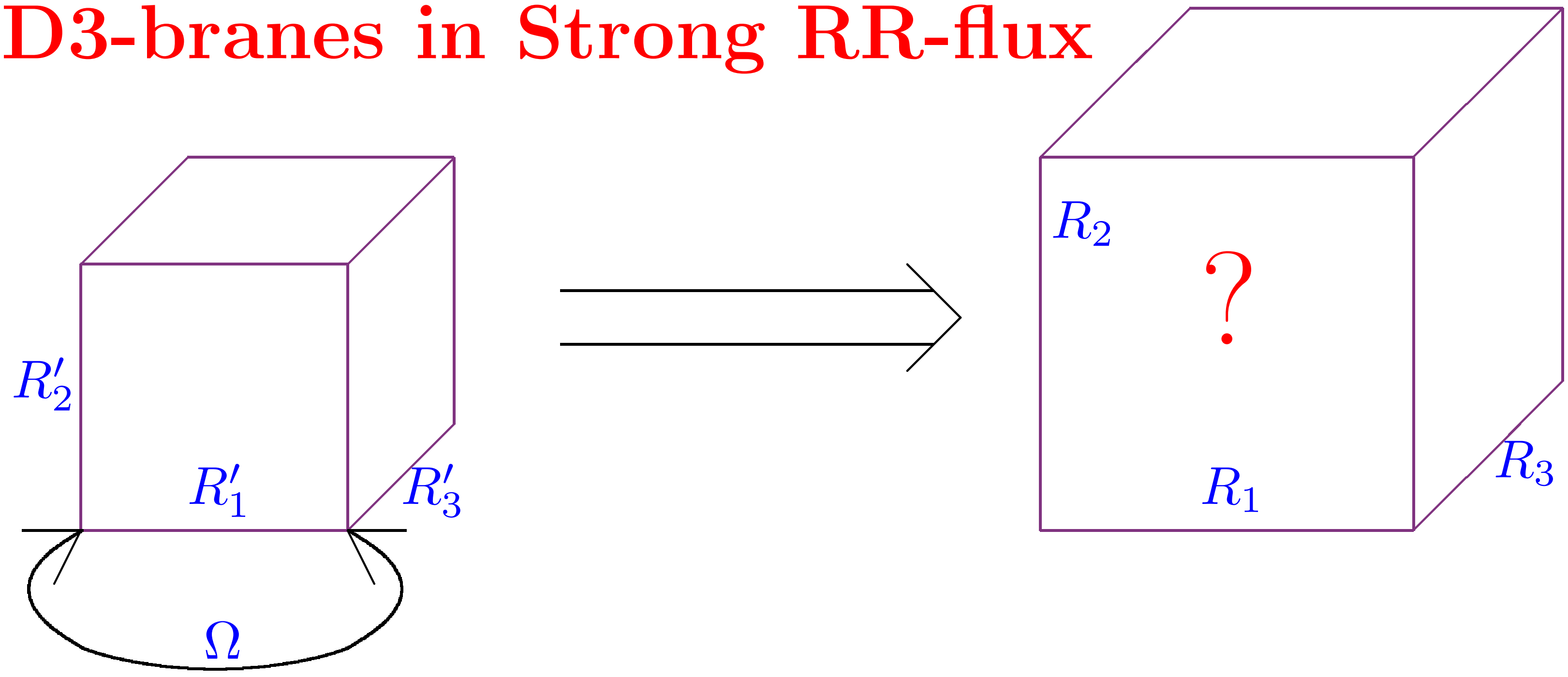
For a particle of mass m:

$$-{\ell_P}^2 m^2 \geq -(4\pi N)^{2/3} {g_s}^2 \beta^{-2/3} \frac{E^2}{\rho^4}.$$

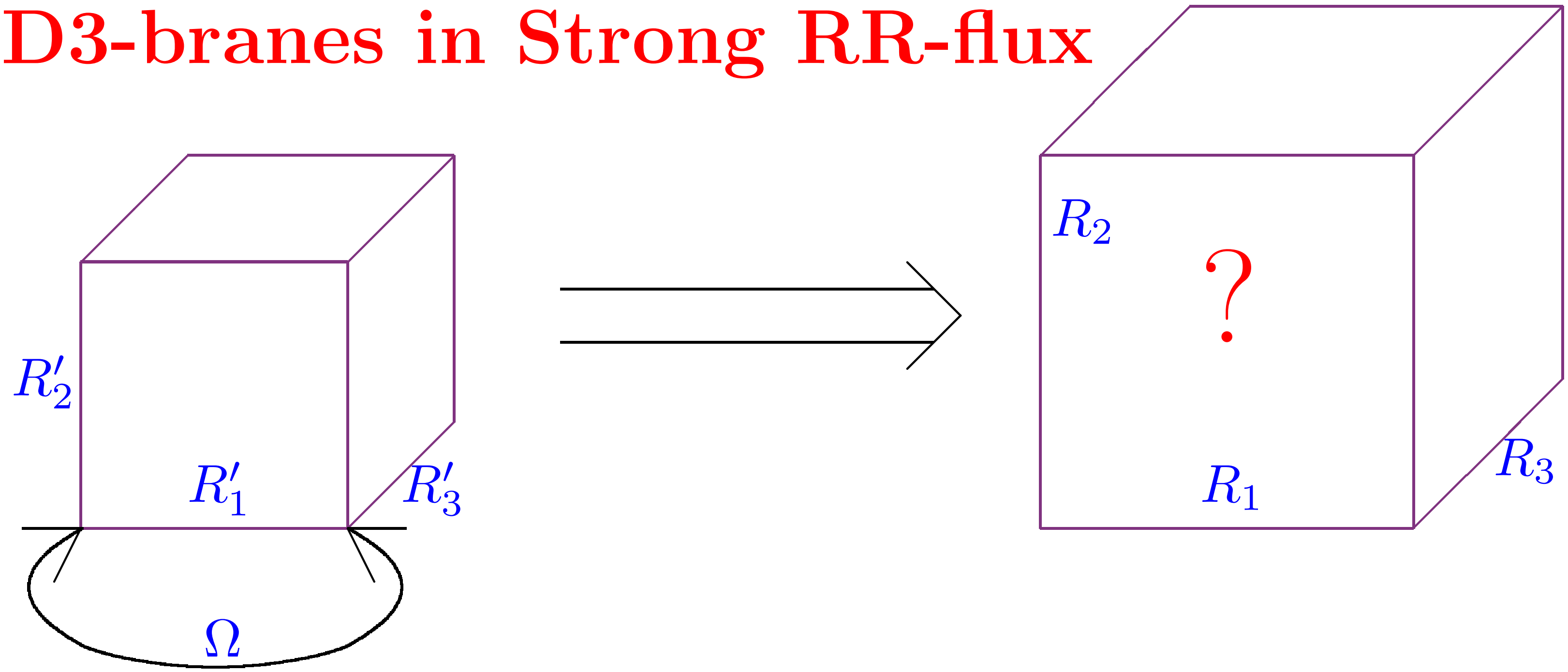
Causality: $$v_{\max} = \sqrt{-\frac{g_{00}}{g_{xx}}} = (4\pi N)^{-1/2} {g_s}^{-3/2} \beta \rho^3.$$

$$\Rightarrow \boxed{E \geq \ell_P m v^{2/3} \beta^{-1/3}}$$

D3-branes in Strong RR-flux



D3-branes in Strong RR-flux



$$f \equiv 1 + \frac{1}{{g_s}^2 \alpha'^4} r^2 \vec{n}^T \zeta^T \zeta \vec{n}$$ [cf. Russo & Tseytlin, 1995]

$$ds^2 = f^{-\frac{1}{2}} \sum_{i=1}^{3} dx_i^2 + f^{\frac{1}{2}} (dr^2 + r^2 d\Omega_5^2 - dt^2) - \frac{f^{-\frac{1}{2}} r^4}{{g_s}^2 \alpha'^4} \left(\vec{n}^T \zeta d\vec{n}\right)^2,$$

$$C_4' = \frac{1}{4{g_s}^2 \alpha'^2} f^{-1} r^2 \vec{n}^T \zeta d\vec{n} \wedge dx_1 \wedge dx_2 \wedge dx_3$$

Summary

- Lorentz violating $SO(3)$-invariant QFT.
- Decoupled from Gravity.
- Nonlocal.
- Characteristic TJ coupling.

Open issues

- Fundamental formulation?
- Theories other than $\mathcal{N} = 4$ SYM?
- Tests?
- Relation to Little-String-Theory?
- cf. M-theory duals of (p, q) 5-branes. [Witten, 1997]

More open issues

- Which objects have $c > 1$?

  cf. nonrelativistic dispersion relations of NCYM solitons [Landsteiner & Lopez & Tytgat, Bak & Lee & Park, Hashimoto & Itzhaki, 2000].

Even more open issues

- Relation to nonassociative structure and nongeometrical twists? [Ramgoolam, Medeiros & Ramgoolam, Bouwknegt & Hannabuss & Mathai, Shelton & Taylor & Wecht, Ellwood & Hashimoto, Nastase, . . . , 2004-6]
- Lightlike limit? cf. Brane probes of limits of Melvin twists. [. . . , Hashimoto & Sethi, Robins & Sethi, Hashimoto & Thomas, Sheikh-Jabbari, Alishahiha & Safarzadeh & Yavartanoo, Gimon & Hashimoto & Hubeny & Lunin & Rangamani, Alishahiha & OG, Varadarajan & OG]