Mass Terms in Twistor String Theory

Ori Ganor (UC Berkeley)

Talk at Princeton University, April 2005.

BASED ON:

- "Massless and massive three dimensional super Yang-Mills theory and mini-twistor string theory," [arXiv:hep-th/0502076]
  by D. W. Chiou, OG, Y. P. Hong, B. S. Kim and I. Mitra.

Background

- Witten discovered remarkable properties of perturbative scattering amplitudes in $N = 4$ SYM in $D = 4$.

- Switching from a basis of plane-waves to a basis of shock-waves (twistors), Witten found that amplitudes vanish unless certain algebraic conditions (on the incoming and outgoing twistors) hold.

- Witten proposed that a topological B-model with target space $\mathbb{CP}^{3|4}$ (super twistor space) reproduces the SYM amplitudes. Certain non-perturbative effects (D1-instantons) are a crucial ingredient.

Further developments

Several further developments:

- Berkovits proposed an alternative (perhaps dual) string theory where worldsheet instantons rather than D1-instantons calculate the amplitudes.

- Aganagic & Vafa found the mirror of the B-model on $\mathbb{CP}^{3|4}$.

- Witten and Berkovits & Motl explained how parity symmetry is restored.

- Bars and Sinkovics & Verlinde derived twistor space from higher-dimensions.

Further developments . . .

- Cachazo & Svrcek & Witten developed a technique for calculating general amplitudes from MHV building blocks.

- Several groups calculated loop amplitudes. [Bern & Dixon & Kosower, Cachazo & Svrcek & Witten, Britto & Cachazo & Feng, . . .]

- Berkovits & Witten showed that in twistor string theory conformal supergravity is coupled to SYM.

- Kulaxizi & Zoubos found marginal deformations of $N = 4$ SYM in twistor string theory.

- Several groups studied orbifolds of twistor string theory. [Park & Rey, Giombi & Kulaxizi & Ricci & Robles-Llana & Trancanelli & Zoubos]

- . . .

Motivation

We would like to add mass terms:

$$g^2\mathcal{L} = \mathrm{tr}\Big\{\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\sum_I D_\mu\Phi^I D^\mu\Phi^I - \frac{1}{4}\sum_{I,J}[\Phi^I,\Phi^J]^2 + \sum_A \psi^A_\alpha \sigma^{\mu\,\alpha\dot\beta}\partial_\mu\psi_{A\dot\beta} + \sum_{A,B,I}\left(\Gamma^I_{AB}\Phi^I\psi^A_\alpha\psi^{B\alpha} + \Gamma^{I\,AB}\Phi^I\psi_{A\dot\alpha}\psi^{\dot\alpha}_B\right) + \sum_{A,B} M_{AB}\psi^A_\alpha\psi^{B\alpha} + \sum_{A,B} M^{AB}\psi_{A\dot\alpha}\psi^{\dot\alpha}_B + (m^2)_{IJ}\Phi^I\Phi^J\Big\},$$

symbol	spacetime	$SU(4)_R$
Φ^I	scalars	$\mathbf{6}$
ψ^A_α	(L-)spinors	$\mathbf{4}$
$\psi_{A\dot\alpha}$	(R-)spinors	$\overline{\mathbf{4}}$
M^{AB}	-	$\mathbf{10}$
M_{AB}	-	$\overline{\mathbf{10}}$

We are then going to dimensionally reduce to $D = 3$ and test the proposal for mass terms. (And we have some puzzles . . .)

Notation

- Shock-waves on $\mathbb{R}^{2,2}$ (or $\mathbb{C}^4$):

$$\boxed{\Phi_{(\mu,\lambda)}(x^1,\ldots,x^4) \propto \delta^2(x_{\alpha\dot\alpha}\lambda^\alpha + \mu_{\dot\alpha})}$$

$$\alpha = 1,2, \quad \dot\alpha = \dot1,\dot2, \quad x_{\alpha\dot\alpha} = x^\mu \sigma_{\mu\alpha\dot\alpha}.$$

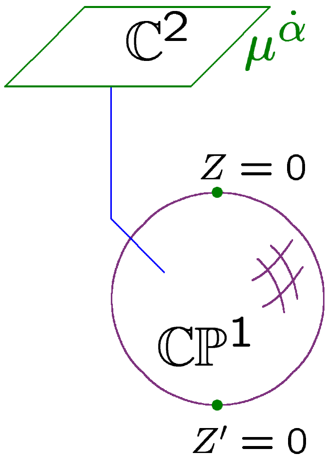
- $\widetilde{t} = (\mu,\lambda)$ denotes a $D=4$ twistor.

- Twistor space is $\mathbb{CP}^3 \setminus \mathbb{CP}^1$ with projective co-ordinates

$$Z^1 = \lambda^1, Z^2 = \lambda^2, Z^3 = \mu^{\dot1}, Z^4 = \mu^{\dot2},$$
$$(Z^1,Z^2,Z^3,Z^4) \sim (\zeta Z^1, \zeta Z^2, \zeta Z^3, \zeta Z^4),$$
$$(Z^1,Z^2) \neq (0,0).$$

Picture of twistor space

$\mathbb{CP}^3 \setminus \mathbb{CP}^1$



$$Z = \lambda^1/\lambda^2,$$

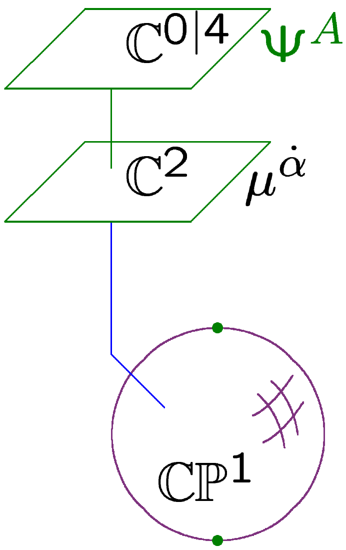
Twistor space is a fibration of $\mathbb{C}^2$ over $\mathbb{CP}^1$.

Two patches:

$\lambda^1 \neq 0$	$\lambda^2 \neq 0$
$Z = \lambda^2/\lambda^1$	$Z' = \lambda^1/\lambda^2 = \frac{1}{Z}$
$X = \mu^{\dot{1}}/\lambda^1$	$X' = \mu^{\dot{1}}/\lambda^2 = \frac{X}{Z}$
$Y = \mu^{\dot{2}}/\lambda^1$	$Y' = \mu^{\dot{2}}/\lambda^2 = \frac{Y}{Z}$

Supertwistor space

For $N = 4$ SYM, Witten added four anticommuting coordinates $\psi^1, \ldots, \psi^4$.



$$\mathbb{CP}^{3|4} \setminus \mathbb{CP}^{1|4}$$

Super-twistor space is a fibration of $\mathbb{C}^{2|4}$ over $\mathbb{CP}^1$.

Two patches:

$\lambda^1 \neq 0$	$\lambda^2 \neq 0$
$Z = \lambda^2/\lambda^1$	$Z' = \lambda^1/\lambda^2 = 1/Z$
$X = \mu^{\dot 1}/\lambda^1$	$X' = \mu^{\dot 1}/\lambda^2 = X/Z$
$Y = \mu^{\dot 2}/\lambda^1$	$Y' = \mu^{\dot 2}/\lambda^2 = Y/Z$
$\Theta^A = \psi^A/\lambda^1$	$\Theta'^A = \psi^A/\lambda^2 = \Theta^A/Z$

Chiral Fermion Mass Term

Claim #1:

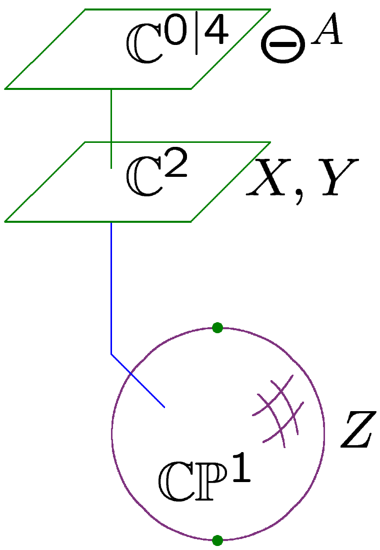
Adding a chiral mass term:

$$\delta\mathcal{L} = \sum_{A,B} M^{AB} \psi_{A\dot{\alpha}} \psi^{\dot{\alpha}}_{B}$$

is equivalent to a certain super-complex structure deformation of supertwistor space $\mathbb{CP}^{3|4} \setminus \mathbb{CP}^{1|4}$.

Note: The chiral mass term breaks CPT, but all we are doing here is summing Feynman diagrams. We don't care about unitarity . . .

The Θ^3 Supercomplex Structure Deformation



$\mathbb{CP}^{3|4} \setminus \mathbb{CP}^{1|4}$

$\lambda^1 \neq 0$	coordinates for $\lambda^2 \neq 0$
Z	$Z' = \frac{1}{Z}$
X	$X' = \frac{X}{Z}$
Y	$Y' = \frac{Y}{Z}$
Θ^A	$\Theta'^A = \frac{1}{Z}\Theta^A + \frac{1}{6Z^2} M^{AB} \epsilon_{BCDE} \Theta^C \Theta^D \Theta^E$

Infinitesimal chiral mass term

Before we prove the claim, note that an infinitesimal chiral mass term would correspond to the infinitesimal vector field

$$\delta\Theta'^A = \frac{1}{6Z^2}\delta M^{AB}\epsilon_{BCDE}\Theta^C\Theta^D\Theta^E$$
$$= \frac{1}{6Z'}\delta M^{AB}\epsilon_{BCDE}\Theta'^C\Theta'^D\Theta'^E$$

This vector field corresponds to a B-model closed string state, and is associated with a mode of the spacetime conformal supergravity field $\overline{E}^{AB}$. [Berkovits & Witten]

It can be checked that this particular mode is Poincaré invariant [in sheaf cohomology], and has conformal dimension $\Delta = 1$.

Mass term as a VEV of a CSUGRA field

We therefore interpret the infinitesimal mass term as a VEV:

$$\boxed{\delta M^{AB} = \left\langle \overline{E}^{AB} \right\rangle}$$

Similarly the anti-chiral mass term can be interpreted as a VEV:

$$\boxed{\delta M_{AB} = \langle E_{AB} \rangle}$$

The couplings $\delta M^{AB} \psi_{A\dot{\alpha}} \psi_B^{\dot{\alpha}}$ and $\delta M_{AB} \psi_\alpha^A \psi^{B\alpha}$ can be compared to formulas of Berkovits & Witten.

But we can also verify the claim for the relation between a mass term and the supercomplex structure deformation directly . . .

Wave-functions with the mass term

In momentum space, the free Dirac equation with a chiral mass term is

$$p_{\alpha\dot{\alpha}}\psi^{\alpha A} = M^{AB}\psi_{\dot{\alpha}B}, \qquad p_{\alpha\dot{\alpha}}\psi^{\dot{\alpha}}_{A} = 0.$$

Like the massless case ($M^{AB} = 0$),

$$p^2 = 0 \qquad \Longrightarrow \qquad p_{\alpha\dot{\alpha}} = \lambda_\alpha\tilde{\lambda}_{\dot{\alpha}}.$$

For the massless case, the general solution is:

$$\psi_{\dot{\alpha}A} = \tilde{\lambda}_{\dot{\alpha}}\widetilde{\varrho}_A(\lambda,\tilde{\lambda}), \qquad \psi^A_\alpha = \lambda_\alpha\varrho^A(\lambda,\tilde{\lambda}).$$

For the massive case, the general solution is:

$$\psi_{\dot{\alpha}A} = \tilde{\lambda}_{\dot{\alpha}}\widetilde{\varrho}_A, \qquad \psi^A_\alpha = \lambda_\alpha\varrho^A + M^{AB}\eta_\alpha\widetilde{\varrho}_B,$$

where η_α is some spinor that satisfies

$$\eta_\alpha\lambda^\alpha = 1.$$

(Note that η_α is not globally defined!)

The twistor transform of the Dirac Wave-functions

As in the massless case, we get the twistor transform by a Fourier transform:

$$\widehat{\varrho}^A(\lambda, \mu) := \int d^2\tilde{\lambda}\, e^{i\mu\tilde{\lambda}} \varrho^A(\lambda, \tilde{\lambda}),$$
$$\widehat{\widetilde{\varrho}}_A(\lambda, \mu) := \int d^2\tilde{\lambda}\, e^{i\mu\tilde{\lambda}} \widetilde{\varrho}_A(\lambda, \tilde{\lambda}).$$

(Following similar steps as in the appendix of Witten's paper . . .) We take the twistor transforms, plug them into the previous expressions, and integrate over λ and $\tilde{\lambda}$ to convert from momentum-space back to coordinate-space.

We perform the λ integral by gauge-fixing

$$(\lambda^1, \lambda^2) \equiv (1, z), \qquad (\eta_1, \eta_2) \equiv (1, 0).$$

Recall that $\eta_\alpha \lambda^\alpha = 1$

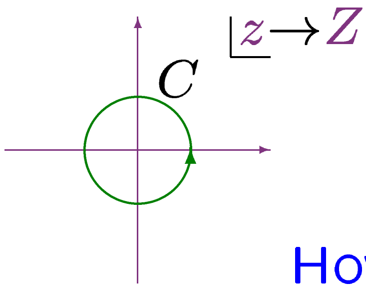
We then convert the λ^2-integral to a z-integral over a path C around the origin.

Contour integrals

We get

$$\overline{\psi}^{\dot\alpha}_A(x) = \frac{1}{2\pi i}\oint_C dz \frac{\partial\widehat{\widetilde{\varrho}}_A}{\partial\mu_{\dot\alpha}}\Big|_{\text{at }(\lambda^\alpha, x_{\alpha\dot\alpha}\lambda^\alpha)}$$

$$\psi^A_\alpha(x) = \frac{1}{2\pi i}\oint_C dz \Big[\lambda_\alpha\widehat{\varrho}^A + M^{AB}\eta_\alpha\widehat{\widetilde{\varrho}}_B\Big]\Big|_{\text{at }(\lambda^\alpha, \underbrace{x_{\alpha\dot\alpha}\lambda^\alpha}_{\mu_{\dot\alpha}})}$$



How do we see that this corresponds to the supercomplex structure deformation

$$\boxed{\Theta'^A = \frac{1}{Z}\Theta^A + \frac{1}{6Z^2}M^{AB}\epsilon_{BCDE}\Theta^C\Theta^D\Theta^E}$$?

Contour integrals . . .

The contour integrals only depend on the residues of the simple poles inside C.

$$\overline{\psi}^{\dot\alpha}_A(x) = \frac{1}{2\pi i}\oint_C dz \partial_{1\dot\alpha}\hat{\tilde{\varrho}}_A(1, z, x_{1\dot 1} + x_{2\dot 1}z, x_{1\dot 2} + x_{2\dot 2}z)$$

(where $\lambda^1 \equiv 1$ and $\lambda^2 \equiv z$)

will not change if we add to $\hat{\tilde{\varrho}}_A$ a function that is holomorphic at $z = \lambda^2/\lambda^1 = 0$.

We can also write the integral as

$$\overline{\psi}^{\dot\alpha}_A(x) = \frac{1}{2\pi i}\oint_C \frac{dz}{z^2}\partial_{2\dot\alpha}\hat{\tilde{\varrho}}_A(\frac{1}{z}, 1, \frac{x_{1\dot 1}}{z} + x_{2\dot 1}, \frac{x_{1\dot 2}}{z} + x_{2\dot 2})$$

(where we replaced $\partial_{1\dot\alpha} \to \partial_{2\dot\alpha}/z$.)

This integral doesn't change if we add to $\hat{\tilde{\varrho}}_A$ a function with at most a simple pole at $z = \lambda^2/\lambda^1 = \infty$ (and an arbitrary singularity at $z = 0$.)

So far, this is just like the massless case [Witten].

What about $\psi^A_\alpha(x)$?

Superfields

The fermion (twistor) fields $\widehat{\widetilde{\varrho}}_A$ and $\widehat{\varrho}^A$ are members of a superfield [Witten]:

$$\mathcal{A}(X, Y, Z, \Theta) =$$
$$\cdots + \widehat{\widetilde{\varrho}}_A \Theta^A + \cdots + \frac{1}{6}\epsilon_{ABCD}\widehat{\varrho}^A \Theta^B \Theta^C \Theta^D + \cdots$$

The contour integrals should be invariant under

$$\mathcal{A} \to \mathcal{A} + (\text{holomorphic at } Z \neq 0) + (\text{holomorphic at } Z \neq \infty)$$

[In other words, $\mathcal{A}$ is an element of sheaf cohomology $H^1(\cdots)$.]

Invariance of the contour integral with the mass term,

$$\psi_\alpha^A(x) = \frac{1}{2\pi i}\oint_C dz \Big[\lambda_\alpha \widehat{\varrho}^A + M^{AB}\eta_\alpha \widehat{\widetilde{\varrho}}_B\Big]\Big|_{\text{at } (\lambda^\alpha, \underbrace{x_{\alpha\dot{\alpha}}\lambda^\alpha}_{\mu_{\dot{\alpha}}})}$$

requires that we define

$$\boxed{\Theta'^A = \frac{1}{Z}\Theta^A + \frac{1}{6Z^2}M^{AB}\epsilon_{BCDE}\Theta^C\Theta^D\Theta^E}$$

as the "good" coordinate near $Z = \infty$.

Holomorphic Curves of Degree $d = 1$

In the undeformed twistor space A holomorphic curve of degree $d = 1$ in $\mathbb{CP}^{3|4}$ is given by a set of linear equations [Witten]

$$X = -x_{1\dot{1}} - x_{2\dot{1}} Z, \qquad Y = -x_{1\dot{2}} - x_{2\dot{2}} Z,$$

$$\Theta^A = -\theta_1^A - \theta_2^A Z,$$

where $x_{\alpha\dot{\alpha}}$ and θ_α^A are moduli.

With the chiral mass term, the last equation has to be replaced with the quadratic expression

$$\boxed{\Theta^A = -\theta_1^A - \theta_2^A Z + M^{AB} \epsilon_{BCDE} \theta_2^C \theta_2^D \theta_2^E Z^2}$$

(In order to have "good" behavior near $Z = \infty$.)

This can be compared with amplitudes ...

Dimensional Reduction

Can we learn more about mass terms by dimensionally reducing to $D = 3$?

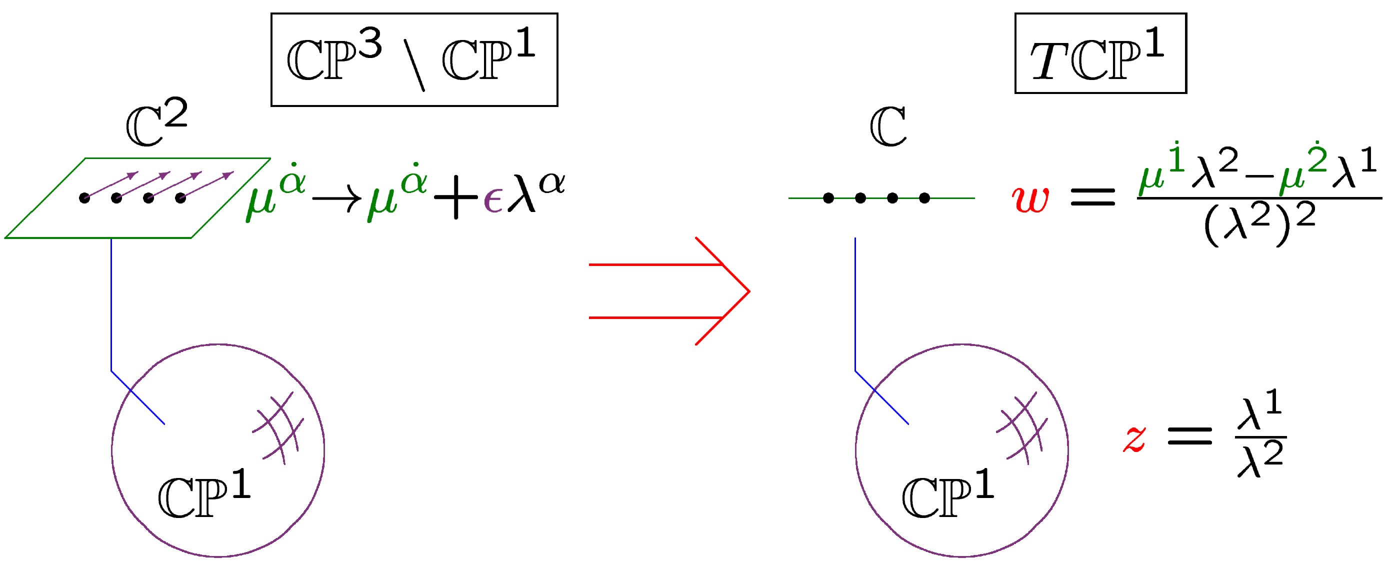
Dimensional Reduction to $D = 3$

We dimensionally reduce to $D = 3$ by gauging the translation generator P_4.

Gauging would make $P_4 = 0$ identically. In an appropriate basis, P_4 acts as

$$\delta\lambda^\alpha = 0, \qquad \delta\mu^{\dot\alpha} = \epsilon\lambda^\alpha.$$

(Note that in $D = 3$ there is no distinction between α and $\dot\alpha$.)



After gauging P_4 we are left with minitwistor space $T\mathbb{CP}^1$. [Hitchin]

Minitwistor space

Minitwistor space is $T\mathbb{CP}^1$ [Hitchin]. It can be parameterized by the P_4-invariant

$$z = \frac{\lambda^1}{\lambda^2}, \qquad w = \frac{\mu^{\dot{1}}\lambda^2 - \mu^{\dot{2}}\lambda^1}{(\lambda^2)^2}$$

For signature $\mathbb{R}^{1,2}$ the minitwistor space is $T\mathbb{CP}^1$ and z, w are real.

The corresponding shock-waves are

$$\phi(x^0, x^1, x^2) = \delta(w + [x^2 - x^0] - 2x^1 z - [x^2 + x^0]z^2)$$

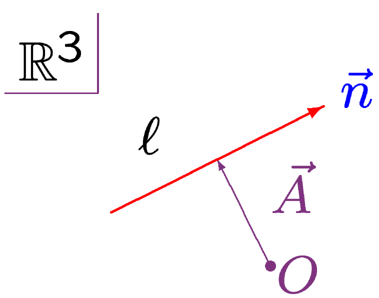
Super-minitwistor space can be covered by two patches with transition relations:

$$z' = \frac{1}{z}, \qquad w' = \frac{1}{z}w, \qquad \theta'^A = \frac{1}{z}\theta^A.$$

Geometrical interpretation

Minitwistor space has a simple geometrical interpretation {that I learned from P. Baird's review}:

$T\mathbb{CP}^1$ is the space of oriented lines in $\mathbb{R}^3$.



$$z = \frac{n^2 - i n^3}{1 + n^1} \in \mathbb{CP}^1 \simeq \mathbb{C} \cup \{\infty\}$$

$$w = \frac{-(1+n^1)(A^2 - iA^3) + (n^2 - in^3)A^1}{(1+n^1)^2}$$

(By stereographic projection.)

Helicity in $D=3$

The Lagrangian of this $D=3$ SYM is

$$g_3^2\mathcal{L} = \mathrm{tr}\Big(\frac{1}{4}F_{ij}F^{ij} + \frac{1}{2}\sum_{i=1}^{7} D_i\Phi^I D^i\Phi^I - \frac{1}{4}\sum_{I,J}[\Phi^I,\Phi^J]^2$$
$$+ \sum_{a=1}^{8}\chi_\alpha^a\sigma^{i\,\alpha\beta}\partial_i\chi_\beta^a + \sum_{a,b,I}\epsilon^{\alpha\beta}\Gamma_{ab}^I\Phi^I\chi_\alpha^a\chi_\beta^b\Big).$$

Onshell, instead of the gauge field we get two scalars:

$$A_4 = \Phi^7, \qquad F_{ij} = \epsilon_{ijl}\partial_l\Phi^8$$

Helicity $\pm$ refers to onshell states with

$$\Phi^7 = \pm i\Phi^8.$$

Holomorphic curves

At tree-level, Witten's discoveries about $D = 4$ SYM amplitudes and holomorphic curves in twistor space immediately imply similar results for $D = 3$.

For example, MHV amplitudes correspond to quadratic sections of $T\mathbb{CP}^1$:

$$w = -[x^2 - x^0] + 2x^1 z + [x^2 + x^0]z^2.$$

For $D = 3$, there is a correspondence [Hitchin]:

holomorphic curves in minitwistor space $T\mathbb{CP}^1$	$\Longleftrightarrow$	Real minimal area surfaces in physical space $\mathbb{R}^3$

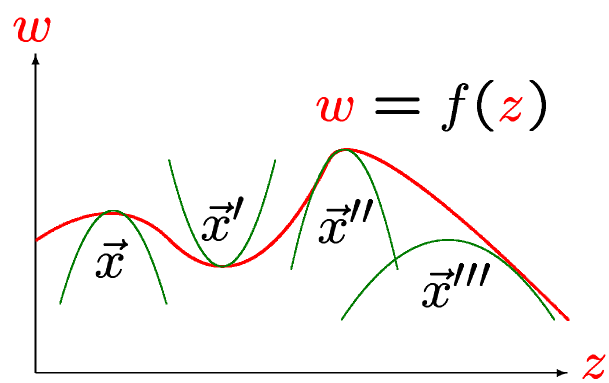
For signature $\mathbb{R}^{1,2}$, this correspondence translates to an amusing physical interpretation for the holomorphic curves.

Algebraic curves in $T\mathbb{RP}^1$ and filaments in $\mathbb{R}^{1,2}$

$$0 = \sum_{r,s} z^r w^s \qquad \text{Expand near } (w_0, z_0) \Longrightarrow$$

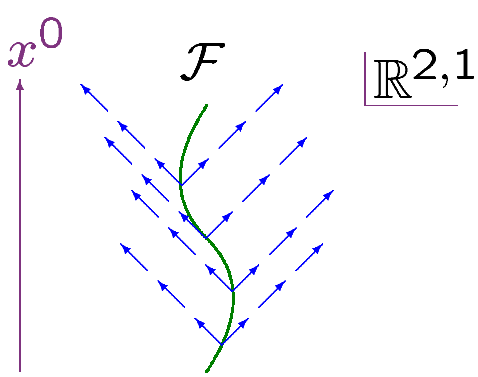
$$w = w_0 + a_1(z - z_0) + a_2(z - z_0)^2 + O(z - z_0)^3$$

We approximate the algebraic curve locally by parabolas. Each parabola corresponds to an MHV curve.



Each parabola therefore corresponds to a point $\vec{x}$ $(\vec{x}', \vec{x}'', \ldots)$ in physical space $\mathbb{R}^{1,2}$. The collection of the points $\vec{x}, \vec{x}', \vec{x}'', \ldots$ forms a filament $\mathcal{F}$.

The filament is a null worldline in $\mathbb{R}^{1,2}$!



The outgoing waves of the scattering process can now be described as a physical disturbance that is emanating from the filament $\mathcal{F}$.

Twisted Dimensional Reduction

We can get $D = 3$ mass terms by gauging a linear combination of translation and $SU(4)$ R-symmetry:

$$P_4 - M^A{}_B R_A{}^B = 0.$$

$R_A{}^B$ is the R-symmetry charge. E.g.,

$$[R_A{}^B, \psi^C] = \delta_A^C \psi^B.$$

For example, Dirac's equation becomes

$$0 = \sum_{\mu=1}^{4} \Gamma^\mu \partial_\mu \psi^A = \sum_{i=1}^{3} \Gamma^i \partial_i \psi^A + i M^A{}_B \psi^B.$$

There is also a mass term for the scalars

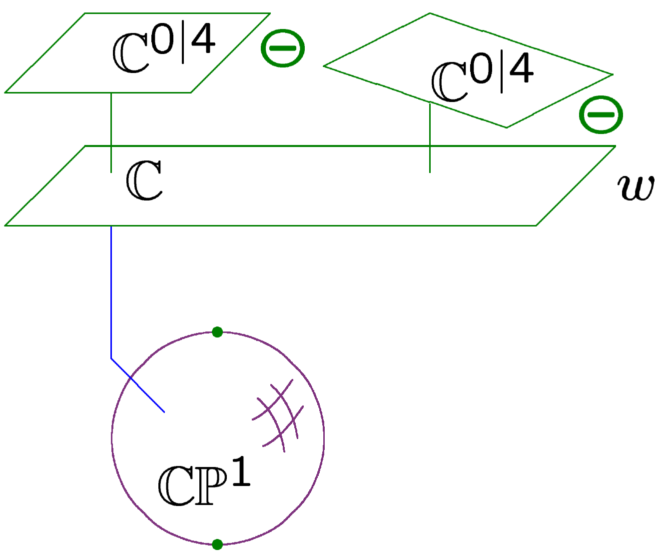
$$0 = \partial^i \partial_i \phi^{[AB]} + M^A{}_C M^B{}_D \phi^{[CD]},$$

Repeating the steps as before, we get instead of minitwistor superspace . . .

$D = 3$ Massive Super-mini-twistor space

Repeating the steps as before, we get the super-mini-twistor target space for the massive $D = 3$ SYM in the form

$$Z' = \frac{1}{Z}, \qquad W' = \frac{W}{Z}, \qquad \Theta' = \frac{1}{Z} \exp\left(\frac{W}{Z} M\right) \Theta$$



The four anticommuting Θ^A directions are fibered in a nontrivial way over the W-plane.

How is this related to direct dimensional reduction of the mass term deformation

$$\Theta'^A = \frac{1}{Z} \Theta^A + \frac{1}{6Z^2} M^{AB} \epsilon_{BCDE} \Theta^C \Theta^D \Theta^E$$

that we found previously?

$D = 3$ Infinitesimal Mass Terms

For infinitesimal mass terms in $D = 3$ we get the following complex structure deformations

$$\delta M^A{}_B \sigma^4_{\alpha\dot\alpha} \psi^{\dot\alpha}_A \psi^{B\alpha} \Longrightarrow \delta\Theta'^A = \delta M^A{}_B \frac{W}{Z^2} \Theta^B,$$

$$\delta M^{AB} \psi^{\dot\alpha}_A \psi_{B\dot\alpha} \Longrightarrow \delta\Theta'^A = \delta M^{AB} \frac{1}{6Z^2} \epsilon_{BCDE} \Theta^C \Theta^D \Theta^E$$

The vector fields on the RHS are the only translationally invariant (in cohomology) $\delta\Theta'$ deformations (unless we allow anticommuting parameters).

The translation generators act as

$$P_1 = iZ \frac{\partial}{\partial W},$$

$$P_+ := P_2 + iP_3 = -i \frac{\partial}{\partial W},$$

$$P_- := P_2 - iP_3 = iZ^2 \frac{\partial}{\partial W}.$$

R-symmetry [Spin(7) in $D = 3$] should transform one mass term to the other. How does Spin(7) act?

Spin(7) R-symmetry

The R-symmetry group of this $D = 3$ SYM is Spin(7). The fields decompose as

$$\psi^A, \psi^A \to \mathbf{4} + \mathbf{4} = \mathbf{8}, \qquad \phi^{[AB]}, \phi^7 \to \mathbf{6} + \mathbf{1} = \mathbf{7}.$$

The adjoint representation of Spin(7) decomposes under the $D = 4$ R-symmetry $SU(4)$ as

$$\mathbf{21} = \mathbf{15} + \mathbf{6}, \qquad \text{generators:} \quad \underbrace{T^A{}_B}_{\mathbf{15}}, \quad \underbrace{T^{AB}}_{\mathbf{6}}$$

The commutation relations are

$$[T^A{}_B, T^C{}_D] = \delta^A_D T^C{}_B - \delta^C_B T^A{}_D,$$
$$[T^A{}_B, T^{CD}] = 2\delta^{[C}_B T^{D]A},$$
$$[T^{AB}, T^{CD}] = 2\epsilon^{EAB[C} T^{D]}{}_E - 2\epsilon^{ECD[A} T^{B]}{}_E$$

Action on the mass operators

We have three mass operators, two of which are vector fields in the B-model:

$$\mathcal{V}_A{}^B := \sigma^4_{\alpha\dot{\alpha}} \psi^{\dot{\alpha}}_A \psi^{B\alpha} \Longrightarrow \frac{W}{Z} \Theta^B \frac{\partial}{\partial \Theta^A},$$

$$\mathcal{V}_{AB} := \psi^{\dot{\alpha}}_A \psi_{B\dot{\alpha}} \Longrightarrow \frac{1}{6Z} \epsilon_{BCDE} \Theta^C \Theta^D \Theta^E \frac{\partial}{\partial \Theta^A}$$

$$\mathcal{V}^{AB} := \psi^{A\alpha} \psi^B_\alpha \Longrightarrow \text{(nonperturbative)}$$

These operators form a Spin(7) irrep that decomposes under the $D = 4$ R-symmetry group $SU(4)$ as

$$\mathbf{35} = \mathbf{15} + \mathbf{10} + \mathbf{10}.$$

The nontrivial R-symmetry generators act as:

$$[T^{AB}, \mathcal{V}_{CD}] = \delta^A_C \mathcal{V}^B{}_D - \delta^B_C \mathcal{V}^A{}_D + \delta^A_D \mathcal{V}^B{}_C - \delta^B_D \mathcal{V}^A{}_C,$$
$$[T^{AB}, \mathcal{V}^{CD}] = \epsilon^{ABCE} \mathcal{V}^D{}_E + \epsilon^{ABDE} \mathcal{V}^C{}_E,$$
$$[T^{AB}, \mathcal{V}^C{}_D] = \epsilon^{ABCE} \mathcal{V}_{ED} + \delta^A_D \mathcal{V}^{BC} - \delta^B_D \mathcal{V}^{AC}$$

Problem

If we could find the action of T^{AB} in twistor string theory, we would learn how to turn on a non-infinitesimal mass term in $D = 4$. (Since we know how to "integrate" $\mathcal{V}^A{}_D$ in $D = 3$.)

[But I only have a very partial answer to this problem.]

Berkovits's model

It might be easier to identify R-symmetry in Berkovits's model.
(cf. parity symmetry [Witten, Berkovits & Motl].)

$$S = \int d^2\mathfrak{z} \Big[\sum_{i=1}^{3} Y_i \nabla_{\bar{\mathfrak{z}}} Z^i + \sum_{A=1}^{4} \Upsilon_A \nabla_{\bar{\mathfrak{z}}} \Psi^A + \text{(right-movers)} \Big] + S_C,$$

$$\nabla_{\bar{\mathfrak{z}}} Z^1 = \partial_{\bar{\mathfrak{z}}} Z^1 - A_{\bar{\mathfrak{z}}} Z^1, \qquad \nabla_{\bar{\mathfrak{z}}} Z^2 = \partial_{\bar{\mathfrak{z}}} Z^2 - A_{\bar{\mathfrak{z}}} Z^2,$$

$$\nabla_{\bar{\mathfrak{z}}} Z^3 = \partial_{\bar{\mathfrak{z}}} Z^3 - 2 A_{\bar{\mathfrak{z}}} Z^3, \qquad \nabla_{\bar{\mathfrak{z}}} \Psi^A = \partial_{\bar{\mathfrak{z}}} \Psi^A - A_{\bar{\mathfrak{z}}} \Psi^A$$

Gauge invariant fields:

$$Z = \frac{Z^1}{Z^2}, \qquad W = \frac{Z^3}{(Z^2)^2}, \qquad \Theta^A = \frac{\Psi^A}{Z^2}.$$

R-symmetry currents in Berkovits's model

Translation currents:

$$\mathcal{P}_+ = -Y_3(Z^1)^2, \quad \mathcal{P}_1 = Y_3 Z^1 Z^2, \quad \mathcal{P}_- = Y_3(Z^2)^2$$

SUSY currents:

$$\mathcal{Q}_{A+} = Z^1 \Upsilon_A, \qquad \overline{\mathcal{Q}}^A_+ = -iY_3 Z^1 \Theta^A,$$
$$\mathcal{Q}_{A-} = Z^2 \Upsilon_A, \qquad \overline{\mathcal{Q}}^A_- = iY_3 Z^2 \Theta^A.$$

$SU(4)$ part of the R-symmetry currents:

$$\mathcal{J}^A{}_B = \Upsilon_B \Theta^A - \frac{1}{4}\delta^A_B \Upsilon_C \Theta^C.$$

Our proposal for the remaining $\mathbf{6}$
R-symmetry currents:

$$\boxed{\mathcal{J}^{AB} = Y_3 \Theta^A \Theta^B + \tfrac{1}{2} Y_3^{-1} \epsilon^{ABCD} \Upsilon_C \Upsilon_D.}$$

Note:
The inverse Y_3^{-1} can be handled by bosonization.

Comments on bosonization

Note that there are two ways to bosonize a a superconformal $\beta\gamma$ ghost system.

Either (i) $\beta = e^{-\phi+\chi}\partial\chi, \quad \gamma = e^{\phi-\chi}$
$\Longrightarrow \gamma^{-1} := e^{-\phi+\chi}, \quad \delta(\gamma) := e^{-\phi}$ no β^{-1} and $\delta(\beta)$.

or (ii) $\beta = e^{-\phi+\chi}, \quad \gamma = e^{\phi-\chi}\partial\chi$
$\Longrightarrow \beta^{-1} := e^{\phi-\chi}, \quad \delta(\beta) = e^{-\phi}$ no γ^{-1} and $\delta(\gamma)$.

[Formulas copied from Polchinski II.]

So, we can have Y_3^{-1} and $\delta(Y_3)$.
[The latter is needed for Berkovits & Motl's instanton number changing operator.]

$\surd$ But we cannot have both Z_3^{-1} and $\delta(Y_3)$.

$\times$ And we cannot have both Z_1^{-1} and $\delta(Y_1)$.

Mass operators in Berkovits's model

A naive application of [Berkovits & Witten]'s rules give

$\mathcal{V}^{AB} := (Z^1 Z^2)^{-1} \Psi^{(A} \partial \Psi^{B)}, \quad$ (from E_{AB})

$\mathcal{V}^{A}{}_{B} := (Z^1 Z^2)^{-1} Z^3 (\Upsilon_B \Psi^A - \frac{1}{4} \delta^A_B \Upsilon_C \Psi^C),$

$\mathcal{V}_{AB} := (Z^1 Z^2)^{-1} \epsilon_{CDE(B} \Upsilon_{A)} \Psi^C \Psi^D \Psi^E \quad$ (from $\overline{E}^{AB}$)

But these do not have the proper commutation relations with the R-symmetry charges T^{AB}. (Calculated from the OPEs with $\mathcal{J}^{AB}$.)

Instead, we found that the following operators are in a Spin(7) mutliplet:

$$
\begin{aligned}
\mathcal{V}^{AB} &= (Z^1 Z^2)^{-1} \Psi^{(A} \partial \Psi^{B)}, \\
\mathcal{V}_{AB} &= (Z^1 Z^2)^{-1} Y_3^{-2} \Upsilon_{(A} \partial \Upsilon_{B)}, \\
\mathcal{V}^{A}{}_{B} &= \tfrac{1}{2} (Z^1 Z^2)^{-1} \Big(Y_3^{-1} \partial \Upsilon_B \Psi^A \\
&\qquad - Y_3^{-1} \Upsilon_B \partial \Psi^A + \partial Y_3^{-1} \Upsilon_B \Psi^A \Big) \\
&\quad - \tfrac{1}{8} (Z^1 Z^2)^{-1} \delta^A_B \Big(Y_3^{-1} \partial \Upsilon_C \Psi^C \\
&\qquad - Y_3^{-1} \Upsilon_C \partial \Psi^C + \partial Y_3^{-1} \Upsilon_C \Psi^C \Big).
\end{aligned}
$$

(Normal ordered.)

Puzzle

Why is the first set of operators not in a Spin(7)-multiplet?

I don't have a good answer, but note that $\mathcal{J}^{AB}$ changes the helicity of the states and hence, in [**?**]'s language, the "instanton-number."

For example, $\mathcal{J}^{AB}$ changes ϕ^7 to $\phi^{[AB]}$ and so

$$\phi^8 = \pm i\phi^7 \Longrightarrow \delta\phi^{AB}.$$

(We get helicity 0 states from helicity ± 1 states.)

Summary

- In $D = 3$ we found that

$$\boxed{Z' = \frac{1}{Z}, \qquad W' = \frac{W}{Z}, \qquad \Theta' = \frac{1}{Z}\,\mathrm{exp}\left(\frac{W}{Z}M\right)\Theta}$$

corresponds to a mass term.

- In $D = 4$ we found that

$$\boxed{\Theta'^{A} = \frac{1}{Z}\Theta^{A} + \frac{1}{6Z^{2}}M^{AB}\epsilon_{BCDE}\Theta^{C}\Theta^{D}\Theta^{E}}$$

corresponds to a chiral mass term.

- In Berkovits's model in $D = 3$ we proposed that

$$\boxed{\mathcal{J}^{AB} = Y_3\Theta^{A}\Theta^{B} + \tfrac{1}{2}Y_3^{-1}\epsilon^{ABCD}\Upsilon_C\Upsilon_D}$$

are the missing Spin(7) R-symmetry currents.

Open issues

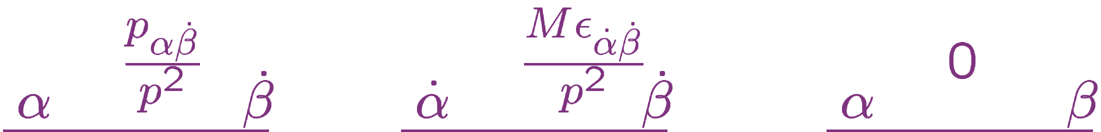
- $D = 3$ at 1-loop?

- The limit $M \to \infty$?

- The Spin(7) R-symmetry . . .

- Mirror symmetry . . .

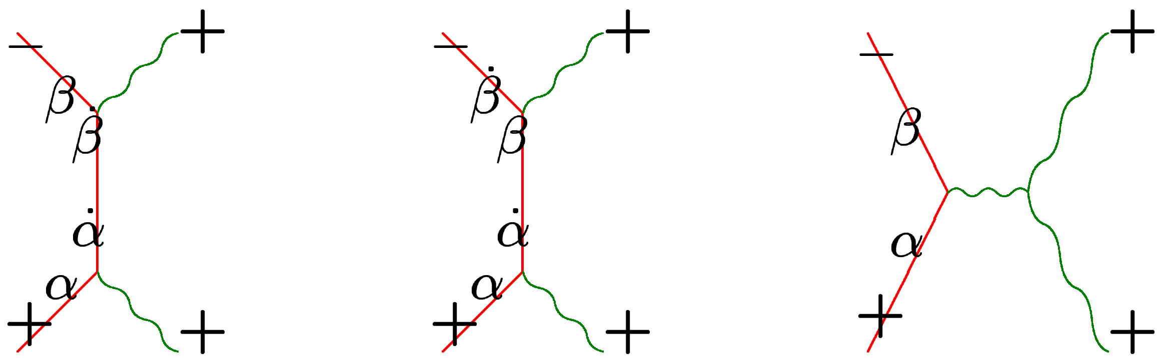
Feynman diagrams with a chiral mass term



The fermion propagator in the presence of a chiral mass term.

The fermion external wavefunction in the presence of a chiral mass term. Here we decomposed the momentum as $p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$ and η_α satisfies $\eta_\alpha \lambda^\alpha = 1$.

Feynman diagrams with a chiral mass term



Planar Feynman diagrams that contribute to the scattering amplitude of helicity (+++-) with a chiral mass term. Wavy lines are gluons and solid lines are fermions, and we use the convention that all external legs are incoming.